ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF BUSINESS, ECONOMICS AND ACCOUNTING

DEPARTMENT OF BUSINESS ADMINISTRATION AND MANAGEMENT

EXAMINATION PAPER

COURSE CODE	:	CBM103
COURSE TITLE	:	QUANTITATIVE ANALYSIS FOR BUSINESS
DURATION	:	3 Hours
LEVEL	:	1.2
DATE	:	

SPECIAL REQUIREMENTS

- Statistical Tables
- Formulae Sheets
- Graph Paper

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets [ ]
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL YOU ARE INSTRUCTED BY THE INVIGILATOR.**

QUESTION ONE

a. Explain the difference between qualitative and quantitative variables. Give practical examples to support your answer. **[5 marks]**

b. A study by the National Park Service revealed that 50% of vacationers going to the Rocky Mountain region visit Yellowstone Park, 40% visit the Tetons, and 35% visit both.

i. What is the probability a vacationer will visit at least one of these attractions? **[2 marks]**

ii. What is the probability 0.35 called? **[1 mark]**

iii. Are the events mutually exclusive? Explain **[2 marks]**

c. Giving practical examples, what are the advantages and disadvantages of stratified sampling method when compared with simple random sampling? **[15 marks]**

QUESTION TWO

a. Your company is in the course of preparing its published accounts and the Chairman has requested that the assets of the Company be compared in a component bar for the last five years. The data for this task are contained in the following table.

Asset	**2013**	**2014**	**2015**	**2016**	**2017**
Property	59	59	65	70	74
Plant and Machinery	176	179	195	210	200
Stock and work-in-progress	409	409	448	516	479
Debtors	330	313	384	374	479
Cash	7	60	29	74	74

i. Construct a suitable chart to present above data **[10 marks]**

ii. Comment upon the movement in the assets over the five-year period. **[5 marks]**

b. Your company manufactures components for use in the production of motor vehicles. The number of components produced each day over a forty-day period is tabulated below:

553	526	521	528	538	523	538	546	524	544
532	554	517	549	512	528	523	510	555	545
524	512	525	543	532	533	519	521	536	534
541	535	531	551	535	519	530	549	518	531

Construct a frequency distribution **[10 marks]**

QUESTION THREE

You are engaged as a managing accounting assistant by the production manager of a light engineering factory, which is investigating the relationship between the number of weeks of experience in wiring of components and the number of components which were rejected as unsatisfactory.

Weeks of experience (x)	4	5	7	9	10	11	12	14
Number of rejects (y)	21	22	15	18	14	14	11	13

As part of the preliminary statistical work, you are asked to do the following:

i. Draw a scatter diagram of the data and explain its meaning. **[5 marks]**

ii. Calculate a coefficient of correlation for these data and interpret its value. **[3 marks]**

iii. Find the least squares regression equation of rejects on experience. **[5 marks]**

iv. Predict using the equation in (iii) above, the number of rejects you would expect from an employee with one week of experience. **[2 marks]**

v. 'Correlation does not prove causation'. Discuss this statement advising the Production Manager in relation to the data above. **[10 marks]**

QUESTION FOUR

a. A debt of \$10000 compounded at 15% is to be discharged over 4 years using a sinking fund method. Find the annual payment (based on ordinary annuity) if the fund earns 11%. Draw up a schedule showing both the position of the debt and the fund each year. **[10 marks]**

b. Given that the demand function is given as $p = 500 - 2q$ and cost function is given as $C = 300q + 2000$

i. Derive the revenue function R **[2 marks]**

ii. Obtain the total profit function **[3 marks]**

iii. How many units per week should be produced in order to maximize profit? **[5 marks]**

iv. Show that the solution of the equation $dR/dq = dC/dq$, where C represents the cost function, gives the same value for q as in part (iii). **[3 marks]**

v. What is the maximum profit available? **[2 marks]**

QUESTION FIVE

a. The director of a state agency believes that the average starting salary for clerical employees in the state is less than \$30000 per year. To test her hypothesis, she has collected a simple random sample of 100 starting clerical salaries from across the state and found that the sample mean is \$29750.

i. State the appropriate null and alternative hypotheses. **[2 marks]**

ii. Assuming the population standard deviation is known to be \$2500 and the significance level for the test is to be 0.05, what is the critical value (stated in dollars)? **[5 marks]**

iii. Referring to your answer in part (ii), what conclusion should be reached with respect to the null hypothesis? **[5 marks]**

iv. Referring to your answer in part (iii), which of the two statistical errors might have been made in this case? Explain. **[3 marks]**

b.. Your company president has told you that the company experiences product returns at the rate of two per month with the number of product returns distributed as a Poisson random variable. Determine the probability that next month there will be:

i. No returns **[3 marks]**

ii. More than two returns **[4 marks]**

iii. At most two returns **[3 marks]**

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

Population mean, $\mu = \frac{\sum x}{N}$

Sample mean, $\bar{x} = \frac{\sum x}{n}$

Median, $M_e = \frac{n+1}{2}$

Grouped data

Population mean, $\mu = \frac{\sum fx}{N}$

Sample mean, $\bar{x} = \frac{\sum fx}{n}$

Median, $Me = L_m + \frac{(\frac{n}{2} - F)C}{fm}$

Mode, $Mo = L_m + \frac{(d_1)\,c}{d_1 + d_2}$

MEASURES OF DISPERSION

Ungrouped data

Population average deviation, $AD = \frac{\sum|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum|x-\bar{x}|}{n}$

Population variance $= \sigma^2 = \frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)}$

Grouped data

Range = maximum value – minimum value

Percentile, $P_k = L_p + \frac{(\frac{kn}{100} - F)C}{fp}$

Decile, $D_x = L_d + \frac{(\frac{xn}{10} - F)C}{fd}$

Lower quartile, $Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{fq}$

Upper quartile $Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{fq}$

Interquartile range $= Q_3 - Q_1$

Quartile deviation $= \frac{Q_3 - Q_1}{2}$

Population average deviation, $AD = \frac{\sum f|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum f|x-\bar{x}|}{n}$

Population variance, $\sigma^2 = \frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, $CV = \frac{\sigma}{\mu} \times 100\%$

Sample coefficient of variation, $CV = \frac{s}{\bar{x}} \times 100\%$

SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, $Sk = \frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, $Sk = \frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis $= \frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis $= = \frac{\sum f(X-\bar{x})^4}{s^4}$

BASIC PROBABILITY CONCEPTS

i. **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

ii. **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

iii. **Complementary rule:** $P(A') = 1 - P(A)$

iv. **Addition rule**

a) If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

b) If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

v. **Multiplication rule**

a) If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

b) If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

vi. **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	**Mean**	**Variance**
Binomial distribution $P(x) = \frac{n!}{x!\,(n-x)!} p^x (q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Standard normal distribution $z = \frac{x-\mu}{\sigma}$	0	1

CONFIDENCE INTERVALS

Mean of a single normal population

i. **If sample size is small (i.e., n<30) and σ^2 Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and σ^2 unknown**

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, σ^2 is known**

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when σ^2 is unknown and sample size is small (n<30), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{X} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H_0 if
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T - distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	Z-distribution T-distribution	$Z > Z_\alpha$ $T > t_\alpha\ (n-1)$
$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	Z-distribution T-distribution	$Z < -Z_\alpha$ $T < -t_\alpha\ (n-1)$

Chi-square tests: $\chi^2_{calc} = \sum \frac{(fo-fe)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$\hat{b}_1 = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$

$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$

Covariance, Cov(X, Y) $= \frac{\sum(XY)}{n} - \bar{X}\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}\right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: Y= a + bX

b $= \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$

$a = \bar{Y} - b\bar{X}$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$\frac{dy}{dx} = \frac{dy}{du} x \frac{du}{dx}$

vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{V^2}$
viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

Simple interest: FV = Principal x Rate x Time

Compound interest: $FV = A(1 + i)^n$

Future Value of an Ordinary Annuity: $FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

Remaining Balance: $B = R\left[\frac{1-(1+i)^{-(n-x)}}{i}\right]$

Net Present Value: $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + \left[\left(\frac{NPV_a}{NPV_a - NPV_b}\right)(b-a)\right]\%$