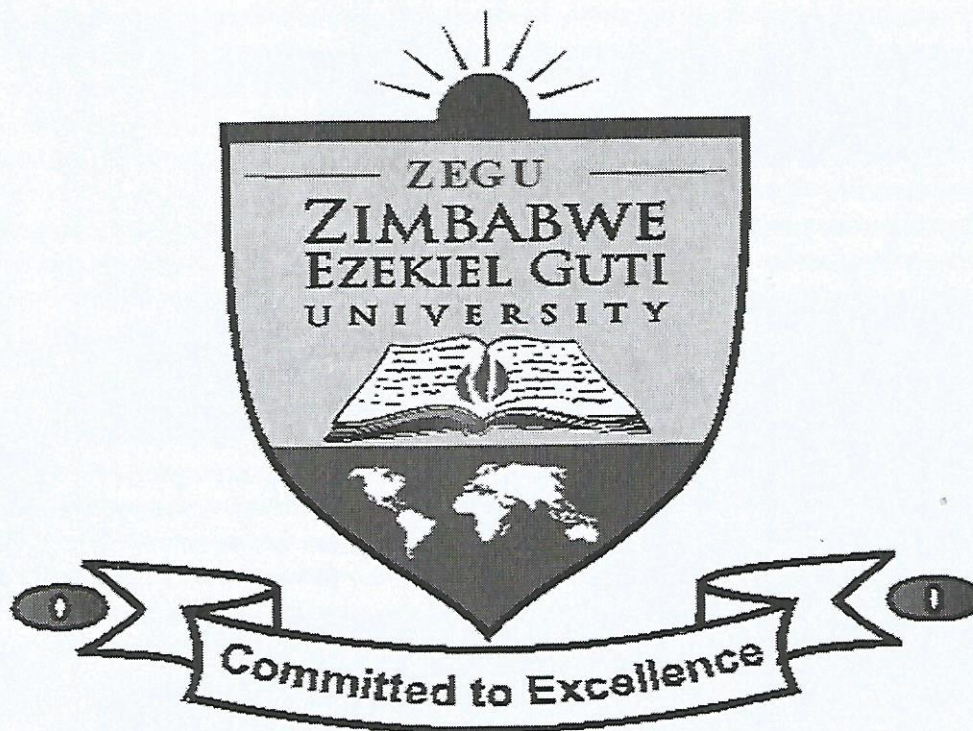


ZIMBABWE EZEKIEL GUTI UNIVERSITY



FACULTY OF HEALTH, SCIENCE AND TECHNOLOGY

Department of Digital Technology

MODULE: MATHEMATICAL CONCEPTS FOR COMPUTING
(MAIN)

CODE: CDT 108

DATE: 31 May 2019

DURATION: 3 HOURS

INSTRUCTIONS AND INFORMATION TO CANDIDATES

ATTEMPT ALL QUESTIONS IN SECTION A

ANSWER ANY **THREE** QUESTIONS IN SECTION B

EACH FULL QUESTION IN SECTION B CARRIES 20 MARKS

NUMBER OF MARKS FOR EACH QUESTION IS GIVEN IN SQUARE BRACKETS []

SHOW ALL WORKING

Section A[40 Marks]: Attempt ALL questions in this section

Question 1

- a) Construct truth tables for each of the following statements. State if the statement is a tautology, contradiction or contingency

i) $(p \vee q) \wedge \sim r$ [3]

ii) $[p \vee (\sim q)] \rightarrow [(\sim p) \wedge q]$ [5]

- b) Evaluate the following argument:

I can enroll for BSIT degree if and only if I pass A' Level Mathematics and any other Science or Commercial subject. Either I pass A' Level Mathematics and a Science subject or A' Level Mathematics and a Commercial subject. I passed A' Level Mathematics and a Science subject. Therefore, I enrolled for BSIT degree. [4]

Question 2

Let

s = All pastors are honest.

t = Some people are fools.

u = I don't like chocolate.

w = The ball is in the sky.

Translate each of the following into English sentences:

a) $(s \wedge \sim t) \rightarrow \sim u$

[2] b) $w \vee (t \wedge \sim u)$

[2]

Question 3

- a) Use the direct proof method to show that the sum of two even integers is even. [5]
- b) Prove that, if n is a positive integer, then the quantity $n^2 + 3n + 2$ is even. [5]

Question 4

The roads represented by the graph in Figure 4 are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights.

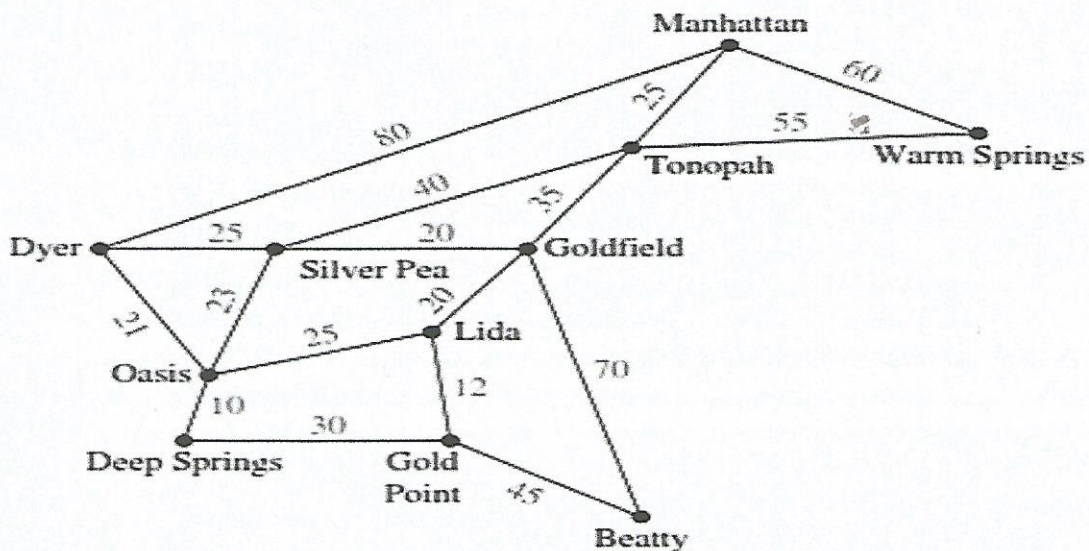


Figure 4: Road Network between towns

Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved? [8]

Question 5

Prove that $1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$. [6]

Section B [60 Marks] Attempt ANY three questions in this section

Question 6

- a) Differentiate *combination* and *permutation*. [4]
- b) What is the probability of
 - i) Getting a six when a die is rolled once? [2]
 - ii) Not getting an even number when a die is rolled once? [2]
 - iii) Not getting an even number when a die is rolled six times? [3]
- c) For each of the following functions, find inverse image of the given set S
 - i) $f: R \rightarrow R, f(x) = 2x, S = \{1\}$ [2]
 - ii) $f: N \rightarrow N \times N, f(x) = (x, x + 1), S = \{(2,2)\}$ [2]
 - iii) $f: R \rightarrow R, f(x) = x, S = \{8\}$ [2]
- d) Find a counterexample for the following statement: $\forall x: Z | x > 0 \vee x < 0$. [1]
- e) State any two methods of proof most appropriate to prove conditional statements [2]

Question 7

- a) Define the term *transitive relation* [1]
- b) How can one identify a *reflexive relation* from a digraph? [2]
- c) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find:
 - i) $A \cup B$
 - ii) $A \cap B$
 - iii) $A - B$
 - iv) $B - A$ [4]
- d) What can you say about the sets A and B if we know the following about them?
 - i) $A \cup B = A$ [2]
 - ii) $A \cap B = A$ [2]
 - iii) $A - B = B - A$ [2]
- e) Suppose the set A represents all tall people, the set B represents all people who play basketball and the set C represents all people that play soccer. For each of the following sets, illustrate them on a Venn diagrams and describe the students in that set.
 - i) $A \cap (B - C)$ [3]
 - ii) $(A \cap B) \cup (A \cap C)$ [4]

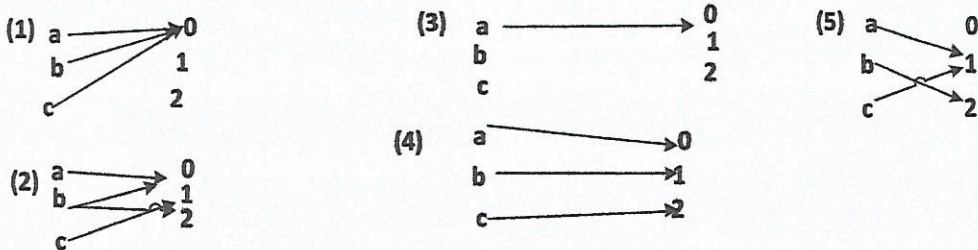
Question 8

Let S and T be relations on a set $A = \{a, b, c, d\}$ where $S = \{(a, a), (a, b), (b, d)\}$ and $T = \{(a, d), (b, c), (b, d), (c, b)\}$.

- a) Determine (*justifying your answer*), if the relations S and T are:
- i) Reflexive,
 - ii) Symmetric,
 - iii) Anti-symmetric,
 - iv) Transitive
- [8]
- b) List the members of the following sets:
- i) $A = \{x | x \in \mathbb{Z} \wedge 3 < x < 12\}$
 - ii) $B = \{x | x = 2 \vee x = 5\}$
- [4]
- c) Prove that $(A - B) \cap (B - A) = \emptyset$
- [8]

Question 9

- a) Consider the following digraphs that represent relations from $A = \{a, b, c\}$ to $B = \{0, 1, 2\}$:



- i) For each relation, state if it is a function or not.
- ii) For those relations that are functions, find the image of the subset $\{a, b\}$.
- iii) For those relations that are not functions, state the properties of a function that are not satisfied.

[10]

b) Suppose that there are five young women and six young men on an island. Each woman is willing to marry some of the men on the island and each man is willing to marry any woman who is willing to marry him. Suppose that Chipo is willing to marry Jason, Larry, and John; Barbara is willing to marry Kevin and Larry; Carol is willing to marry Jason, Nick, and Oscar; Diane is willing to marry Jason, Larry, Nick, and Oscar; and Elizabeth is willing to marry Jason and John.

- i) Model the possible marriages on the island using a bipartite graph. [3]
- ii) Find a matching of the young women and the young men on the island such that each young woman is matched with a young man whom she is willing to marry. [5]
- iii) With justification, state whether the matching you found in part (ii) is a complete and/or maximum matching. [2]

*****End of Exam*****