ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS

DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

EXAMINATION PAPER

MODULE CODE	:	CEC 114
MODULE TITLE	:	STATISTICS FOR ECONOMIST 1
SPECIAL REQUIREMENTS	:	Statistical Tables, Formulae Sheets
DURATION	:	3 Hours
LEVEL	:	1.1
DATE	:	28 NOV 2024

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue
2. Use of silent, non-programmable calculators is allowed
3. Answer **ALL** questions
4. Begin each question on a new page in section B.
5. The number of marks for each question or part question is shown in brackets [ ]
6. Show all workings, where applicable

QUESTION ONE

The following data represent the income distribution for informal sector workers in Zimbabwe for a particular year.

38, 42, 35, 40, 16, 23, 64, 31, 38, 64, 12, 17, 30, 31, 38, 36, 66, 44, 31, 81, 53, 60, 75, 25, 79, 17, 15, 61, 64, 48, 10, 85, 69, 36, 38, 16, 47, 64, 40, 55, 23, 73, 52, 45, 81

a) Construct the frequency distribution. **[5 marks]**

b) Present the data on a Histogram and use the presentation to estimate the modal income. **[5 marks]**

c) Compute the mean, mode, median and coefficient of variation. **[8 marks]**

d)Write a brief report commenting on the statistics computed in (c) above and provide strategic recommendations to the Government of Zimbabwe? **[7 marks]**

QUESTION TWO

(a) With the aid of examples, explain the importance of statistics to an economist working in a health facility. **[10Marks]**

(b) The demand and supply functions of a good are given by:

$$P = -5QD + 160$$

$$P = QS + 9$$

(c) Where P, QD and QS denote the price, quantity demanded and quantity supplied respectively.

(d) Calculate the equilibrium price and quantity. **[5Marks]**

(e) Calculate the new equilibrium price and quantity after the imposition of a fixed tax of $13 per good. **[5Marks]**

(f) Explain the importance of the arithmetic mean in data analysis **[5Marks]**

QUESTION THREE

Land distribution in a newly resettled area is as given below

Plot size (Hectares)	Number of families
Less than 0.5	10
0.5<1.0	40
1.0<1.5	30
1.5<2.0	10
2.0<2.5	7
2.5 and above	3

(a) Estimate the mean and mode **[8Marks]**

(b) Calculate the standard deviation **[5Marks]**

(c) Sketch a histogram **[6Marks]**

(d) If you were an officer in the Ministry of Agriculture, what would be your recommendations to the authorities and why? **[6Marks]**

QUESTION FOUR

You are given the following data for potatoes at a farm and costs of production over the past five years.

Year	Output in thousand units (x)	Cost in thousand dollars (y)
1	20	82
2	16	70
3	24	90
4	22	85
5	18	73

There is high degree of correlation between output and costs, and so it is decided to calculate fixed costs and the variable costs per unit of output using the least squares method.

i. Plot the data and comment. **[4Marks]**

ii. Find an equation to determine the expected level of costs, for any given volume of output and interpret the coefficients. **[8 marks]**

iii. Compute the coefficient of determination and comment **[4 marks]**

iv. Determine the level of cost when output is120 **[4 Marks]**

v. Explain the importance of correlation coefficient in analysis **[5Marks]**

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

Population mean, $\mu = \frac{\sum x}{N}$

Sample mean, $\bar{x} = \frac{\sum x}{n}$

Median, $M_e = \frac{n+1}{2}$

Grouped data

Population mean, $\mu = \frac{\sum fx}{N}$

Sample mean, $\bar{x} = \frac{\sum fx}{n}$

Median, Me = $L_m + \frac{(\frac{n}{2} - F)c}{fm}$

Mode, Mo = $L_m + \frac{(d_1)\, c}{d_1 + d_2}$

MEASURES OF DISPERSION

Ungrouped data

Population average deviation, AD= $\frac{\sum|x-\mu|}{N}$

Sample average deviation, AD = $\frac{\sum|x-\bar{x}|}{n}$

Population variance = $\sigma^2 = \frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)$

Sample variance = $s^2 = \frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)}$

Grouped data

Range = maximum value – minimum value

Percentile, $P_k = L_p + \frac{(\frac{kn}{100} - F)C}{fp}$

Decile, $D_x = L_d + \frac{(\frac{xn}{10} - F)C}{fd}$

Lower quartile, $Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{fq}$

Upper quartile $Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{fq}$

Interquartile range = $Q_3 - Q_1$

Quartile deviation = $\frac{Q_3 - Q_1}{2}$

Population average deviation, AD= $\frac{\sum f|x-\mu|}{N}$

Sample average deviation, AD = $\frac{\sum f|x-\bar{x}|}{n}$

Population variance, $\sigma^2 = \frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)$

Sample variance= $s^2 = \frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, CV= $\frac{\sigma}{\mu} \times 100\%$

Sample coefficient of variation, CV = $\frac{s}{\bar{x}} \times 100\%$

SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, Sk = $\frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, Sk = $\frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis = $\frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis = = $\frac{\sum f(X-\bar{x})^4}{s^4}$

BASIC PROBABILITY CONCEPTS

a) **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

b) **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

c) **Complementary rule:** $P(A') = 1 - P(A)$

d) **Addition rule**

i. If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

ii. If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

e) **Multiplication rule**

i. If X, Y are dependent (conditional probability),

$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$

ii. If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

f) **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	Mean	Variance
Binomial distribution $P(x) = \frac{n!}{x!\,(n-x)!} p^x (q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	**λ**	**λ**
Standard normal distribution $Z = \frac{x - \mu}{\sigma}$	**0**	**1**

CONFIDENCE INTERVALS

Mean of a single normal population

i. **If sample size is small (i.e., n<30) and σ^2 Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \quad \text{or simply} \quad \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and σ^2 unknown**

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}} \quad \text{or simply,} \quad \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, σ^2 is known**

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \text{or simply,} \quad \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when σ^2 is unknown and sample size is small ($n<30$), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H_0 if
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T - distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \le \mu_0$ $H_1: \mu > \mu_0$	Z-distribution T-distribution	$Z > Z_\alpha$ $T > t_\alpha (n-1)$
$H_0: \mu \ge \mu_0$ $H_1: \mu < \mu_0$	Z-distribution T-distribution	$Z < -Z_\alpha$ $T < -t_\alpha (n-1)$

Chi-square tests: $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, $Cov(X, Y) = \frac{\sum(XY)}{n} - \bar{X}\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}\right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: $Y = a + bX$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

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vi. Rule 5: The product rule. If y = uv then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

vii. Rule 6: The quotient rule. If y = $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{V^2}$

viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

Simple interest: FV = Principal x Rate x Time

Compound interest: $FV = A(1 + i)^n$

Future Value of an Ordinary Annuity: $FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

Remaining Balance: $B = R\left[\frac{1-(1+i)^{-(n-x)}}{i}\right]$

Net Present Value: $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + \left[\left(\frac{NPV_a}{NPV_a - NPV_b}\right)(b-a)\right]\%$