ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS

DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

EXAMINATION PAPER

MODULE CODE	:	CBM122
MODULE TITLE	:	Quantitative Analysis for Business
SPECIAL REQUIREMENTS	:	Statistical Tables Formulae Sheets Graph Paper
DURATION	:	3 Hours
LEVEL	:	1.2
DATE	:	29 JUL 2025

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets [ ]
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR INSTRUCTS YOU.**

QUESTION ONE

The speed of cars recorded at a speed trap in Marondera by a detector between 1500 hours and 1700hours on a Public Holiday was summarized in the dataset below:

58, 65, 71, 82, 88, 97, 103, 107, 112, 116, 118, 92, 73, 84, 55, 59, 61, 69, 77, 81, 89, 96, 100, 105, 110, 114, 119, 120, 90, 72

a. Construct the frequency distribution. **[5 marks]**

b. Present the data on a Histogram and use the presentation to estimate the modal speed. **[5 marks]**

c. Compute the mean, mode, median and coefficient of variation. **[8 marks]**

d. Write a brief report commenting on the statistics computed in (c) above and provide strategic recommendations to the Police Superintendent for Marondera? **[7 marks]**

QUESTION TWO

a. Monthly distance travelled by salesmen is approximately normally distributed with mean 20 miles and standard deviation 4 miles. The sales manager considers that salesmen who travel less than 12 miles in one month are performing poorly. The manager wishes to identify the number of miles travelled in one month, above, which only 1% of salesmen are expected to exceed. What monthly mileage is this? **[5 marks]**

b. According to a survey published by ComPsych Corporation, 54% of all workers read e-mail while they are talking on the phone. Suppose that 20% of those who read e-mail while they are talking on the phone write personal "to-do" lists during meetings. Assuming that these figures are true for all workers, if a worker is randomly selected, determine the probability that the worker does not write personal "to-do" lists given that he reads e-mail while talking on the phone. **[5 marks]**

c. Your company uses a machine in its production department, which costs $44000 at the beginning of 2018. The machine will be replaced after five years usage by a new machine at the end of 2023. During the five years of operation of the machine it is estimated that the net cash inflows at the beginning of each year will be as follows:

Year	2019	2020	2021	2022	2023
Net cash inflow ($)	26000	30000	50000	(25000)	(12000)

If the $44000 debt, which is compounded semi-annually at 6%, is to be discharged in 2023 by a sinking fund method, under which annual deposits will be made into a fund paying 4% annually, produce the schedule for the sinking fund. **[15 marks]**

QUESTION THREE

a. If a company employs 3500 people and if a random sample of 175 of these employees has been taken by systematic sampling:

i. What is the value of k? **[2 marks]**

ii. The researcher would start the sample selection between what two values? **[2 marks]**

iii. Where could the researcher obtain a frame for this study? **[1 mark]**

b. According to the Bureau of Labor Statistics, the average weekly earnings of a production worker in 2017 were $424.20. Suppose a labor researcher wants to test to determine whether this figure is still accurate today. The researcher randomly selects 54 production workers from across the country and obtains a representative earnings statement for one week from each. The resulting sample average is $432.69. Assuming a population standard deviation of $33.90, and a 5% level of significance, determine whether the mean weekly earnings of a production worker have changed. **[20 marks]**

QUESTION FOUR

a. Is it possible to predict the annual number of business bankruptcies by the number of firm births (business starts) in the country? The following data published by the Small Business Administration, are pairs of the number of business bankruptcies and the number of firm births for a six-year period.

Business Bankruptcies	Firm Births
34.3	58.1
35.0	55.4
38.5	57.0
40.1	58.5
35.5	57.4
37.9	58.0

i. Using a scatter plot, comment on the relationship between business bankruptcies by the number of firm births. **[4 marks]**

ii. Use these data to develop the equation of the regression model to predict the number of business bankruptcies by the number of firm births. **[6 marks]**

iii. Discuss the slope and *y*-intercept of the model. **[2 marks]**

iv. Compute the coefficient of determination and comment. **[4 marks]**

v. Compute the correlation coefficient and comment. **[4 marks]**

b. The following table shows the number of insurance policies by class of business issued by an insurance company during the years 2019-2023.

Policy type	**Number in years**				
	2019	**2020**	**2021**	**2022**	**2023**
Life	24	27	32	31	33
Motor	42	37	31	29	26
Household	10	14	21	28	35
Other	7	5	8	7	4

Carefully draw a suitable chart to illustrate the data. **[5 marks]**

QUESTION FIVE

a. Jedidiah is buying a machine costing \$120000 for a small-scale business. In order to run it he will spend \$85000 in the first year, \$30000 in the second, \$15000 in the third year and a further \$15000 in the fourth year. Revenues expected from the business are \$80000, \$120000, \$100000 and \$65000 in the 1st, 2nd, 3rd, 4th years respectively. If the cost of capital is at 10% per annum, advise Jedidiah as to whether the business is viable. **[10 marks]**

b. Your firm has recently started to give economic advice to your clients. Acting as a consultant you have estimated the demand curve of a client's firm to be:

$P = 200 - 8x$, where x is output and P is price.

Investigation of the client firm's cost (TC) profile shows that total cost is given by:

$$TC = \frac{1}{3}x^3 - 14x^2 + 211x + 10$$

i. Using the methods of differentiation, find the turning points of the firm's profit curve and say whether these point(s) are maxima or minima. **[10 marks]**

ii. Calculate the maximum profit. **[5 marks]**

THE END

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

Population mean, $\mu = \frac{\sum x}{N}$

Sample mean, $\bar{x} = \frac{\sum x}{n}$

Median, $M_e = \frac{n+1}{2}$

Grouped data

Population mean, $\mu = \frac{\sum fx}{N}$

Sample mean, $\bar{x} = \frac{\sum fx}{n}$

Median, $Me = L_m + \frac{(\frac{n}{2} - F)C}{fm}$

Mode, $Mo = L_m + \frac{(d_1)\,c}{d_1 + d_2}$

MEASURES OF DISPERSION

Ungrouped data

Population average deviation, $AD = \frac{\sum |x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum |x-\bar{x}|}{n}$

Population variance $= \sigma^2 = \frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)}$

Grouped data

Range = maximum value – minimum value

Percentile, $P_k = L_p + \frac{(\frac{kn}{100} - F)C}{fp}$

Decile, $D_x = L_d + \frac{(\frac{xn}{10} - F)C}{fd}$

Lower quartile, $Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{fq}$

Upper quartile $Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{fq}$

Interquartile range = $Q_3 - Q_1$

Quartile deviation = $\frac{Q_3 - Q_1}{2}$

Population average deviation, AD= $\frac{\sum f|x-\mu|}{N}$

Sample average deviation, AD = $\frac{\sum f|x-\bar{x}|}{n}$

Population variance, $\sigma^2 = \frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)$

Sample variance= $s^2 = \frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, CV= $\frac{\sigma}{\mu}$ x 100%

Sample coefficient of variation, CV = $\frac{s}{\bar{x}}$ x 100%

SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, Sk = $\frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, Sk = $\frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis = $\frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis = = $\frac{\sum f(X-\bar{x})^4}{s^4}$

BASIC PROBABILITY CONCEPTS

a) **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

b) **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

c) **Complementary rule:** $P(A') = 1 - P(A)$

d) **Addition rule**

i. If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

ii. If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

e) **Multiplication rule**

i. If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

ii. If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

f) **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	Mean	Variance
Binomial distribution $P(x) = \frac{n!}{x!\,(n-x)!} p^x(q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Standard normal distribution $z = \frac{x-\mu}{\sigma}$	0	1

CONFIDENCE INTERVALS

Mean of a single normal population

i. **If sample size is small (i.e., n<30) and σ^2 Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and σ^2 unknown**

$$\bar{x} - Z_{\alpha/2}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm Z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, σ^2 is known**

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when σ^2 is unknown and sample size is small (n<30), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H_0 if
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T – distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \le \mu_0$ $H_1: \mu > \mu_0$	Z-distribution T-distribution	$Z > Z_\alpha$ $T > t_\alpha (n-1)$
$H_0: \mu \ge \mu_0$ $H_1: \mu < \mu_0$	Z-distribution T-distribution	$Z < -Z_\alpha$ $T < -t_\alpha (n-1)$

Chi-square tests: $\chi^2_{calc} = \sum \frac{(fo-fe)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_I X$

$$\hat{b}_I = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_I \bar{X}$$

Covariance, Cov(X, Y) $= \frac{\sum(XY)}{n} - \bar{X}\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}\right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: Y= a + bX

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$
viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

Simple interest: FV = Principal x Rate x Time

Compound interest: $FV = A(1 + i)^n$

Future Value of an Ordinary Annuity: $FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

Remaining Balance: $B = R\left[\frac{1-(1+i)^{-(n-X)}}{i}\right]$

Net Present Value: $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + \left[\left(\frac{NPV_a}{NPV_a - NPVb}\right)(b-a)\right] \%$