



ZIMBABWE EZEKIEL GUTI UNIVERSITY
FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS
DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

EXAMINATION PAPER

MODULE CODE : CBM122
MODULE TITLE : Quantitative Analysis for
Business
SPECIAL REQUIREMENTS : Statistical Tables
Formulae Sheets
Graph Paper
DURATION : 3 Hours
LEVEL : 1.2
DATE : 10 APR 2025

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets []
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR INSTRUCTS YOU.**

QUESTION ONE

A random sample of 30 customer records for a physician's office showed the following time (in days) to collect insurance payments:

Number of Days to Collect Payment

34 55 36 39 36 32 35 30 47 31 60 66 48
43 33 24 37 38 65 35 22 45 33 29 41 38
35 28 56 56

- Construct the frequency distribution. [6 marks]
- Present the data on a Histogram [5 marks]
- Use the presentation in (b) to estimate the modal number of days. [2 marks]
- Compute the mean, mode, median and coefficient of variation. [8 marks]
- Write a brief report commenting on the statistics computed above and provide strategic recommendations to the physician's office? [4 marks]

QUESTION TWO

- The demand and supply functions of a good are given by:

$$P = -4Q_D + 120$$

$$P = \frac{1}{3}Q_S + 29$$

Where P , Q_D and Q_S denote the price, quantity demanded and quantity supplied respectively.

- Calculate the equilibrium price and quantity. [5 marks]
 - Calculate the new equilibrium price and quantity after the imposition of a fixed tax of \$13 per good. [5 marks]
 - Who pays the tax? [5 marks]
- According to the Labor Statistics, 75% of the women 25 through 49 years of age participate in the labor force. Suppose 78% of the women in that age group are married. Suppose also that 61% of all women 25 through 49 years of age are married and are participating in the labor force. What is the probability that a randomly selected woman in that age group is neither married nor participating in the labor force? [5 marks]
 - In the past few years, outsourcing overseas has become more frequently used than ever before by Zimbabwean companies. However, outsourcing is not without

problems. A recent survey by *Purchasing* indicates that 20% of the companies that outsource overseas use a consultant. Suppose 15 companies that outsource overseas are randomly selected. What is the probability that between four and seven (inclusive) companies that outsource overseas use a consultant? **[5 marks]**

QUESTION THREE

a. According to a study several years ago by the Personal Communications Industry Association, the average wireless phone user earns \$62,600 per year. Suppose a researcher believes that the average annual earnings of a wireless phone user are lower now, and he sets up a study in an attempt to prove his theory. He randomly samples 18 wireless phone users and finds out that the average annual salary for this sample is \$58,974, with a population standard deviation of \$7810. Use $\alpha = 0.01$ to test the researcher's theory. Assume wages in this industry are normally distributed. **[20 marks]**

b. According to a report by Ruzivo Research, the average monthly household cellular phone bill is \$60. Suppose local monthly household cell phone bills are normally distributed with a standard deviation of \$11.35. What is the probability that a randomly selected monthly cell phone bill is between \$45 and \$70? **[5 marks]**

QUESTION FOUR

People in the aerospace industry believe the cost of a space project is a function of the weight of the major object being sent into space.

Weight (tons)	Cost (\$ millions)
1.897	53.6
3.019	184.9
0.453	6.4
0.988	23.5
1.058	33.4
2.100	110.4
2.387	104.6

As part of the preliminary statistical work, you are asked to do the following:

- Draw a scatter diagram of the data and explain its meaning. **[5 marks]**
- Find the least squares regression equation to predict the cost of a space project by the weight of the space object. **[6 marks]**

- c. Predict using the equation in (ii) above, the cost of a space you would expect from weight of the space object with four tons. [2 marks]
- d. Calculate a correlation coefficient for these data and interpret its value. [4 marks]
- e. Calculate a coefficient of determination for these data and interpret its value. [4 marks]
- f. 'Correlation does not prove causation'. Discuss this statement advising the People in the aerospace industry in relation to the data above. [4 marks]

QUESTION FIVE

You borrow USD\$48000 from a financial institution for purposes of paying for your tuition and accommodation fees. If Interest is charged at the rate of 11% compounded monthly for a 2-year period.

- a. Calculate the periodic. [5 marks]
- b. Construct the relevant amortization schedule. [17 marks]
- c. How much is outstanding after the 7th payment? [1 mark]
- d. What are the Interest and Capital portions of the 14th payment? [2 marks]

THE END

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

$$\text{Population mean, } \mu = \frac{\sum x}{N}$$

$$\text{Sample mean, } \bar{x} = \frac{\sum x}{n}$$

$$\text{Median, } M_e = \frac{n+1}{2}$$

Grouped data

$$\text{Population mean, } \mu = \frac{\sum fx}{N}$$

$$\text{Sample mean, } \bar{x} = \frac{\sum fx}{n}$$

$$\text{Median, } M_e = L_m + \frac{(\frac{n}{2} - F)C}{f_m}$$

$$\text{Mode, } M_o = L_m + \frac{(d_1)C}{d_1 + d_2}$$

MEASURES OF DISPERSION

Ungrouped data

$$\text{Population average deviation, } AD = \frac{\sum |x - \mu|}{N}$$

$$\text{Sample average deviation, } AD = \frac{\sum |x - \bar{x}|}{n}$$

$$\text{Population variance} = \sigma^2 = \frac{1}{N} (\sum x^2 - \frac{1}{N} (\sum x)^2)$$

$$\text{Sample variance} = s^2 = \frac{1}{n-1} (\sum x^2 - \frac{1}{n} (\sum x)^2)$$

$$\text{Population standard deviation, } \sigma = \sqrt{\frac{1}{N} (\sum x^2 - \frac{1}{N} (\sum x)^2)}$$

$$\text{Sample standard deviation, } s = \sqrt{\frac{1}{n-1} (\sum x^2 - \frac{1}{n} (\sum x)^2)}$$

Grouped data

Range = maximum value – minimum value

$$\text{Percentile, } P_k = L_p + \frac{(\frac{kn}{100} - F)C}{f_p}$$

$$\text{Decile, } D_x = L_d + \frac{(\frac{xn}{10} - F)C}{f_d}$$

$$\text{Lower quartile, } Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{f_q}$$

$$\text{Upper quartile } Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{f_q}$$

Interquartile range = $Q_3 - Q_1$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2}$$

$$\text{Population average deviation, AD} = \frac{\sum f|x - \mu|}{N}$$

$$\text{Sample average deviation, AD} = \frac{\sum f|x - \bar{x}|}{n}$$

$$\text{Population variance, } \sigma^2 = \frac{1}{N} (\sum fx^2 - \frac{1}{N} (\sum fx)^2)$$

$$\text{Sample variance} = s^2 = \frac{1}{n-1} (\sum fx^2 - \frac{1}{n} (\sum fx)^2)$$

$$\text{Population standard deviation, } \sigma = \sqrt{\frac{1}{N} (\sum fx^2 - \frac{1}{N} (\sum fx)^2)}$$

$$\text{Sample standard deviation, } s = \sqrt{\frac{1}{n-1} (\sum fx^2 - \frac{1}{n} (\sum fx)^2)}$$

$$\text{Population coefficient of variation, CV} = \frac{\sigma}{\mu} \times 100\%$$

$$\text{Sample coefficient of variation, CV} = \frac{s}{\bar{x}} \times 100\%$$

SHAPE OF FREQUENCY DISTRIBUTIONS

$$\text{Population skewness, Sk} = \frac{3(\mu - \text{median})}{\sigma} \text{ or } \frac{(\mu - \text{mode})}{\sigma}$$

$$\text{Sample skewness, Sk} = \frac{3(\bar{x} - \text{median})}{s} \text{ or } \frac{(\bar{x} - \text{mode})}{s}$$

$$\text{Population kurtosis} = \frac{\sum f(X - \mu)^4}{\sigma^4}$$

$$\text{Sample kurtosis} = \frac{\sum f(X - \bar{x})^4}{s^4}$$

BASIC PROBABILITY CONCEPTS

a) **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

b) **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

c) **Complementary rule:** $P(A') = 1 - P(A)$

d) **Addition rule**

i. If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

ii. If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

e) **Multiplication rule**

i. If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

ii. If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

f) **Law of Conditional Probability,** $P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y|X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	Mean	Variance
Binomial distribution $P(x) = \frac{n!}{x!(n-x)!} p^x(q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Standard normal distribution $Z = \frac{x - \mu}{\sigma}$	0	1

CONFIDENCE INTERVALS

Mean of a single normal population

- i. If sample size is small (i.e., $n < 30$) and σ^2 Unknown

$$\bar{x} - t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}}$$

- ii. If sample size is large and σ^2 unknown

$$\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

- iii. If variance, σ^2 is known

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

- i. If when σ^2 is unknown and sample size is small ($n < 30$), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_k}{\frac{s}{\sqrt{n}}}$$

- ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_k}{\frac{\sigma}{\sqrt{n}}} = Z = \frac{\bar{x} - \mu_k}{\frac{\sigma}{\sqrt{n}}}$$

- iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random

variable such that: $z = \frac{\bar{x} - \mu_k}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H_0 if
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T - distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	Z-distribution	$Z > Z_{\alpha}$
	T-distribution	$T > t_{\alpha} (n-1)$
$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	Z-distribution	$Z < -Z_{\alpha}$
	T-distribution	$T < -t_{\alpha} (n-1)$

Chi-square tests: $\chi^2_{\text{calc}} = \sum \frac{(f_o - f_e)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, $\text{Cov}(X, Y) = \frac{\sum(XY)}{n} - \bar{X} \bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n \sum XY - \sum X \sum Y}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n \sum XY - \sum X \sum Y}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}} \right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: $Y = a + bX$

$$b = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b \bar{X}$$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

- i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
- ii. Rule 1: The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c .
- iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
- iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
- v. Rule 4: The chain rule. If y is a function of u , which is itself a function of x , then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
- vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
- viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{f_x}{f_y}$

Simple interest:

$$FV = \text{Principal} \times \text{Rate} \times \text{Time}$$

Compound interest:

$$FV = A(1+i)^n$$

Future Value of an Ordinary Annuity:

$$FV = R \left[\frac{(1+i)^n - 1}{i} \right]$$

Regular deposit:

$$R = \frac{FVi}{(1+i)^n - 1}$$

Present Value of an Ordinary Annuity: $PV = R \left[\frac{1-(1+i)^{-n}}{i} \right]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

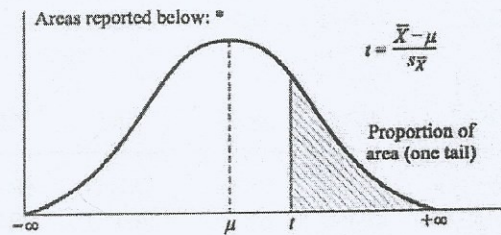
Remaining Balance: $B = R \left[\frac{1-(1+i)^{-(n-x)}}{i} \right]$

Net Present Value: $NPV = \sum_{j=0}^n \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + \left[\frac{NPV_a}{NPV_a - NPV_b} \right] (b-a) \%$

APPENDIX 5

Student's *t* Distribution



Proportions of Area for the *t* Distributions

<i>df</i>	0.10	0.05	0.025	0.01	0.005	<i>df</i>	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657	18	1.330	1.734	2.101	2.552	2.878
2	1.886	2.920	4.303	6.965	9.925	19	1.328	1.729	2.093	2.539	2.861
3	1.638	2.353	3.182	4.541	5.841	20	1.325	1.725	2.086	2.528	2.845
4	1.533	2.132	2.776	3.747	4.604	21	1.323	1.721	2.080	2.518	2.831
5	1.476	2.015	2.571	3.365	4.032	22	1.321	1.717	2.074	2.508	2.819
6	1.440	1.943	2.447	3.143	3.707	23	1.319	1.714	2.069	2.500	2.807
7	1.415	1.895	2.365	2.998	3.499	24	1.318	1.711	2.064	2.492	2.797
8	1.397	1.860	2.306	2.896	3.355	25	1.316	1.708	2.060	2.485	2.787
9	1.383	1.833	2.262	2.821	3.250	26	1.315	1.706	2.056	2.479	2.779
10	1.372	1.812	2.228	2.764	3.169	27	1.314	1.703	2.052	2.473	2.771
11	1.363	1.796	2.201	2.718	3.106	28	1.313	1.701	2.048	2.467	2.763
12	1.356	1.782	2.179	2.681	3.055	29	1.311	1.699	2.045	2.462	2.756
13	1.350	1.771	2.160	2.650	3.012	30	1.310	1.697	2.042	2.457	2.750
14	1.345	1.761	2.145	2.624	2.977	40	1.303	1.684	2.021	2.423	2.704
15	1.341	1.753	2.131	2.602	2.947	60	1.296	1.671	2.000	2.390	2.660
16	1.337	1.746	2.120	2.583	2.921	120	1.289	1.658	1.980	2.358	2.617
17	1.333	1.740	2.110	2.567	2.898	∞	1.282	1.645	1.960	2.326	2.576

*Example: For the shaded area to represent 0.05 of the total area of 1.0, value of *t* with 10 degrees of freedom is 1.812
 Source: From Table III of Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, 6th ed., 1974, published by Longman Group Ltd., London (previously published by Oliver & Boyd, Edinburgh), by permission of the authors and publishers.

Present Value Table

Present value of 1 i.e. $(1 + r)^{-n}$

Where r = discount rate
 n = number of periods until payment

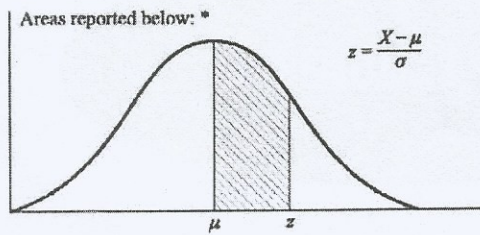
Periods (n)	Discount rate (r)										
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	1
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	2
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	3
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683	4
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	5
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	6
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	7
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	8
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424	9
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386	10
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	11
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	12
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	13
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	14
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	15
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694	2
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579	3
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482	4
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402	5
6	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335	6
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279	7
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233	8
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194	9
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162	10
11	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135	11
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112	12
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093	13
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078	14
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.074	0.065	15

Annuity Table

Present value of an annuity of 1 i.e. $\frac{1 - (1 + r)^{-n}}{r}$

Where r = discount rate
 n = number of periods

Periods (n)	Discount rate (r)										
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	1
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	2
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	3
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	4
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	5
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	6
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	7
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	8
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	9
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	10
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	11
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	12
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103	13
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367	14
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606	15
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	2
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	3
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	4
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	5
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326	6
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	7
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	8
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	9
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192	10
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	11
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	12
13	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533	13
14	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611	14
15	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675	15



Proportions of Area for the Standard Normal Distribution

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4014
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987									
3.5	.4997									
4.0	.4999									

*Example: For $z = 1.96$, shaded area is 0.4750 out of the total area of 1.0000.

APPENDIX 1

Binomial Distribution

<i>n</i>	<i>x</i>	.01	.05	.10	.15	.20	.25	<i>p</i>	.30	.35	.40	.45	.50
1	0	.9900	.9500	.9000	.8500	.8000	.7500		.7000	.6500	.6000	.5500	.5000
	1	.0100	.0500	.1000	.1500	.2000	.2500		.3000	.3500	.4000	.4500	.5000
2	0	.9801	.9025	.8100	.7225	.6400	.5625		.4900	.4225	.3600	.3025	.2500
	1	.0198	.0950	.1800	.2550	.3200	.3750		.4200	.4550	.4800	.4950	.5000
	2	.0001	.0025	.1100	.0225	.0400	.0625		.0900	.1225	.1600	.2025	.2500
3	0	.9703	.8574	.7290	.6141	.5120	.4219		.3430	.2746	.2160	.1664	.1250
	1	.0294	.1354	.2430	.3251	.3840	.4219		.4410	.4436	.4320	.4084	.3750
	2	.0003	.0071	.0270	.0574	.0960	.1406		.1890	.2289	.2880	.3341	.3750
	3	.0000	.0001	.0010	.0034	.0080	.0156		.0270	.0429	.0640	.0911	.1250
4	0	.9606	.8145	.6561	.5220	.4096	.3164		.2401	.1785	.1296	.0915	.0625
	1	.0388	.1715	.2916	.3685	.4096	.4219		.4116	.3845	.3456	.2995	.2500
	2	.0006	.0135	.0486	.0975	.1536	.2109		.2646	.3105	.3456	.3675	.3750
	3	.0000	.0005	.0036	.0115	.0256	.0469		.0756	.1115	.1536	.2005	.2500
	4	.0000	.0000	.0001	.0005	.0016	.0039		.0081	.0150	.0256	.0410	.0625
5	0	.9510	.7738	.5905	.4437	.3277	.2373		.1681	.1160	.0778	.0503	.0312
	1	.0480	.2036	.3280	.3915	.4096	.3955		.3602	.3124	.2592	.2059	.1562
	2	.0010	.0214	.0729	.1382	.2048	.2637		.3087	.3364	.3456	.3369	.3125
	3	.0000	.0011	.0081	.0244	.0512	.0879		.1323	.1811	.2304	.2757	.3125
	4	.0000	.0000	.0004	.0022	.0064	.0146		.0284	.0488	.0768	.1128	.1562
	5	.0000	.0000	.0000	.0001	.0003	.0010		.0024	.0053	.0102	.0185	.0312
6	0	.9415	.7351	.5314	.3771	.2621	.1780		.1176	.0754	.0467	.0277	.0156
	1	.0571	.2321	.3543	.3993	.3932	.3560		.3025	.2437	.1866	.1359	.0938
	2	.0014	.0305	.0984	.1762	.2458	.2966		.3241	.3280	.3110	.2780	.2344
	3	.0000	.0021	.0146	.0415	.0819	.1318		.1852	.2355	.2765	.3032	.3125
	4	.0000	.0001	.0012	.0055	.0154	.0330		.0595	.0951	.1382	.1861	.2344
	5	.0000	.0000	.0001	.0004	.0015	.0044		.0102	.0205	.0369	.0609	.0938
	6	.0000	.0000	.0000	.0000	.0001	.0002		.0007	.0018	.0041	.0083	.0156
7	0	.9321	.6983	.4783	.3206	.2097	.1335		.0824	.0490	.0280	.0152	.0078
	1	.0659	.2573	.3720	.3960	.3670	.3115		.2471	.1848	.1306	.0872	.0547
	2	.0020	.0406	.1240	.2097	.2753	.3115		.3177	.2985	.2613	.2140	.1641

n	x	.01	.05	.10	.15	.20	.25	p	.30	.35	.40	.45	.50
3		.0000	.0036	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734	
4		.0000	.0002	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734	
5		.0000	.0000	.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641	
6		.0000	.0000	.0000	.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547	
7		.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0016	.0037	.0078	
8	0	.9227	.6634	.4305	.2725	.1678	.1002	.0576	.0319	.0168	.0084	.0039	
	1	.0746	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0312	
	2	.0026	.0515	.1488	.2376	.2936	.3115	.2065	.2587	.2090	.1569	.1094	
	3	.0001	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188	
	4	.0000	.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734	
	5	.0000	.0000	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188	
	6	.0000	.0000	.0000	.0002	.0011	.0038	.0100	.0217	.0413	.0403	.1094	
	7	.0000	.0000	.0000	.0000	.0001	.0004	.0012	.0033	.0079	.0164	.0312	
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0017	.0039	
9	0	.9135	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020	
	1	.0830	.2985	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176	
	2	.0034	.0629	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703	
	3	.0001	.0077	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641	
	4	.0000	.0006	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461	
	5	.0000	.0000	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461	
	6	.0000	.0000	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641	
	7	.0000	.0000	.0000	.0000	.0003	.0012	.0039	.0098	.0212	.0407	.0703	
	8	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0013	.0035	.0083	.0176	
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0008	.0020	
10	0	.9044	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010	
	1	.0914	.3151	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098	
	2	.0042	.0746	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439	
	3	.0001	.0105	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172	
	4	.0000	.0010	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051	
	5	.0000	.0001	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461	
	6	.0000	.0000	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051	
	7	.0000	.0000	.0000	.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172	
	8	.0000	.0000	.0000	.0000	.0001	.0004	.0014	.0043	.0106	.0229	.0439	
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0016	.0042	.0098	
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	
11	0	.8953	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014	.0005	
	1	.0995	.3293	.3835	.3248	.2363	.1549	.0932	.0518	.0266	.0125	.0054	
	2	.0050	.0867	.2131	.2866	.2953	.2581	.1998	.1395	.0887	.0513	.0269	
	3	.0002	.0137	.0710	.1517	.2215	.2581	.2568	.2254	.1774	.1259	.0806	
	4	.0000	.0014	.0158	.0536	.1107	.1721	.2201	.2428	.2365	.2060	.1611	
	5	.0000	.0001	.0025	.0132	.0388	.0803	.1321	.1830	.2207	.2360	.2256	
	6	.0000	.0000	.0003	.0023	.0097	.0268	.0566	.0985	.1471	.1931	.2256	
	7	.0000	.0000	.0000	.0003	.0017	.0064	.0173	.0379	.0701	.1128	.1611	
	8	.0000	.0000	.0000	.0000	.0002	.0011	.0037	.0102	.0234	.0462	.0806	
	9	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018	.0052	.0126	.0269	
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0007	.0021	.0054	
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0005	
12	0	.8864	.5404	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002	
	1	.1074	.3413	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029	
	2	.0060	.0988	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161	
	3	.0002	.0173	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537	
	4	.0000	.0021	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208	
	5	.0000	.0002	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934	
	6	.0000	.0000	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256	

<i>n</i>	<i>x</i>	.01	.05	.10	.15	.20	.25	<i>p</i>	.30	.35	.40	.45	.50
13	7	.0000	.0000	.0000	.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934	
	8	.0000	.0000	.0000	.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208	
	9	.0000	.0000	.0000	.0000	.0001	.0004	.0015	.0048	.0125	.0277	.0537	
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0025	.0068	.0161	
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0029
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
	0	.8775	.5133	.2542	.1209	.0550	.0238	.0097	.0037	.0013	.0004	.0001	.0000
	1	.1152	.3512	.3672	.2774	.1787	.1029	.0540	.0259	.0113	.0045	.0016	.0006
	2	.0070	.1109	.2448	.2937	.2680	.2059	.1388	.0836	.0453	.0220	.0095	.0034
	3	.0003	.0214	.0997	.1900	.2457	.2517	.2181	.1651	.1107	.0660	.0349	.0161
	4	.0000	.0028	.0277	.0838	.1535	.2097	.2337	.2222	.1845	.1350	.0873	.0495
	5	.0000	.0003	.0055	.0266	.0691	.1258	.1803	.2154	.2214	.1989	.1571	.1095
	6	.0000	.0000	.0008	.0063	.0230	.0559	.1030	.1546	.1968	.2169	.2095	.1775
7	.0000	.0000	.0001	.0011	.0058	.0186	.0442	.0833	.1312	.1775	.2095	.1833	
8	.0000	.0000	.0001	.0001	.0011	.0047	.0142	.0336	.0656	.1089	.1571	.2095	
9	.0000	.0000	.0000	.0000	.0001	.0009	.0034	.0101	.0243	.0495	.0873	.1350	
10	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0022	.0065	.0162	.0349	.0537	
11	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0012	.0036	.0095	.0161	
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0016	.0034	
13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	
14	0	.8687	.4877	.2288	.1028	.0440	.0178	.0068	.0024	.0008	.0002	.0001	.0000
	1	.1229	.3593	.3559	.2539	.1539	.0832	.0467	.0181	.0073	.0027	.0009	.0003
	2	.0081	.1229	.2570	.2912	.2501	.1802	.1134	.0634	.0317	.0141	.0056	.0020
	3	.0003	.0259	.1142	.2056	.2501	.2402	.1943	.1366	.0845	.0462	.0222	.0095
	4	.0000	.0037	.0349	.0998	.1720	.2202	.2290	.2022	.1549	.1040	.0611	.0349
	5	.0000	.0004	.0078	.0352	.0860	.1468	.1963	.2178	.2066	.1701	.1222	.0873
	6	.0000	.0000	.0013	.0093	.0322	.0734	.1262	.1759	.2066	.2088	.1833	.1350
	7	.0000	.0000	.0002	.0019	.0092	.0280	.0618	.1082	.1574	.1952	.2095	.1775
	8	.0000	.0000	.0000	.0003	.0020	.0082	.0232	.0510	.0918	.1398	.1833	.1350
	9	.0000	.0000	.0000	.0000	.0003	.0018	.0066	.0183	.0408	.0762	.1222	.1611
	10	.0000	.0000	.0000	.0000	.0000	.0003	.0014	.0049	.0136	.0312	.0611	.0916
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0010	.0033	.0093	.0222	.0495
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0019	.0056	.0104
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0009	.0020
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	
15	0	.8601	.4633	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	.0000	.0000
	1	.1303	.3658	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005	.0001
	2	.0092	.1348	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032	.0010
	3	.0004	.0307	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139	.0056
	4	.0000	.0049	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417	.0161
	5	.0000	.0006	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916	.0537
	6	.0000	.0000	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527	.1095
	7	.0000	.0000	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964	.1489
	8	.0000	.0000	.0000	.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964	.1489
	9	.0000	.0000	.0000	.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527	.1964
	10	.0000	.0000	.0000	.0000	.0001	.0007	.0030	.0096	.0245	.0515	.0916	.1350
	11	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0074	.0191	.0417	.0762
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0016	.0052	.0139	.0349
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0032	.0095
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0020
15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
16	0	.8515	.4401	.1853	.0743	.0281	.0100	.0033	.0010	.0003	.0001	.0000	.0000
	1	.1376	.3706	.3294	.2097	.1126	.0535	.0228	.0087	.0030	.0009	.0002	.0001
	2	.0104	.1463	.2745	.2775	.2111	.1336	.0732	.0353	.0150	.0056	.0018	.0006

<i>n</i>	<i>x</i>	.01	.05	.10	.15	.20	.25	<i>p</i>	.30	.35	.40	.45	.50
	3	.0005	.0359	.1423	.2285	.2463	.2079	.1465	.0888	.0468	.0215	.0085	
	4	.0000	.0061	.0514	.1311	.2001	.2252	.2040	.1553	.1014	.0572	.0278	
	5	.0000	.0008	.0137	.0555	.1201	.1802	.2099	.2008	.1623	.1123	.0667	
	6	.0000	.0001	.0028	.0180	.0550	.1101	.1649	.1982	.1983	.1684	.1222	
	7	.0000	.0000	.0004	.0045	.0197	.0524	.1010	.1524	.1889	.1969	.1746	
	8	.0000	.0000	.0001	.0009	.0055	.0197	.0487	.0923	.1417	.1812	.1964	
	9	.0000	.0000	.0000	.0001	.0012	.0058	.0185	.0442	.0840	.1318	.1746	
	10	.0000	.0000	.0000	.0000	.0002	.0014	.0056	.0167	.0392	.0755	.1222	
	11	.0000	.0000	.0000	.0000	.0000	.0002	.0013	.0049	.0142	.0337	.0667	
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0040	.0115	.0278	
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0029	.0085	
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018	
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
17	0	.8429	.4181	.1668	.0631	.0225	.0075	.0023	.0007	.0002	.0000	.0000	.0000
	1	.1447	.3741	.3150	.1893	.0957	.0426	.0169	.0060	.0019	.0005	.0001	.0000
	2	.0117	.1575	.2800	.2673	.1914	.1136	.0581	.0260	.0102	.0035	.0010	.0000
	3	.0006	.0415	.1556	.2359	.2393	.1893	.1245	.0701	.0341	.0144	.0052	.0000
	4	.0000	.0076	.0605	.1457	.2093	.2209	.1868	.1320	.0796	.0411	.0182	.0000
	5	.0000	.0010	.0175	.0668	.1361	.1914	.2081	.1849	.1379	.0875	.0472	.0000
	6	.0000	.0001	.0039	.0236	.0680	.1276	.1784	.1991	.1839	.1432	.1000	.0000
	7	.0000	.0000	.0007	.0065	.0267	.0668	.1201	.1685	.1927	.1841	.1484	.0000
	8	.0000	.0000	.0001	.0014	.0084	.0279	.0644	.1134	.1606	.1883	.1555	.0000
	9	.0000	.0000	.0000	.0003	.0021	.0093	.0276	.0611	.1070	.1540	.1855	.0000
	10	.0000	.0000	.0000	.0000	.0004	.0025	.0095	.0263	.0571	.1008	.1484	.0000
	11	.0000	.0000	.0000	.0000	.0001	.0005	.0026	.0090	.0242	.0525	.0944	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0081	.0215	.0472	.0000
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0021	.0068	.0182	.0000
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0016	.0052	.0000
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0000
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0000
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
18	0	.8345	.3972	.1501	.0536	.0180	.0056	.0016	.0004	.0001	.0000	.0000	.0000
	1	.1517	.3763	.3002	.1704	.0811	.0338	.0126	.0042	.0012	.0003	.0001	.0000
	2	.0130	.1683	.2835	.2556	.1723	.0958	.0458	.0190	.0069	.0022	.0006	.0000
	3	.0007	.0473	.1680	.2406	.2297	.1704	.1046	.0547	.0246	.0095	.0031	.0000
	4	.0000	.0093	.0700	.1592	.2153	.2130	.1681	.1104	.0614	.0291	.0117	.0000
	5	.0000	.0014	.0218	.0787	.1507	.1988	.2017	.1664	.1146	.0666	.0327	.0000
	6	.0000	.0002	.0052	.0301	.0816	.1436	.1873	.1941	.1655	.1181	.0708	.0000
	7	.0000	.0000	.0010	.0091	.0350	.0820	.1376	.1792	.1892	.1657	.1214	.0000
	8	.0000	.0000	.0002	.0022	.0120	.0376	.0811	.1327	.1734	.1864	.1669	.0000
	9	.0000	.0000	.0000	.0004	.0033	.0139	.0386	.0794	.1284	.1694	.1855	.0000
	10	.0000	.0000	.0000	.0001	.0008	.0042	.0149	.0385	.0771	.1248	.1669	.0000
	11	.0000	.0000	.0000	.0000	.0001	.0010	.0046	.0151	.0374	.0742	.1214	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0047	.0145	.0354	.0708	.0000
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0045	.0134	.0327	.0000
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0039	.0117	.0000
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0009	.0031	.0000
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0000
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0000
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
19	0	.8262	.3774	.1351	.0456	.0144	.0042	.0011	.0003	.0001	.0000	.0000	.0000
	1	.1586	.3774	.2852	.1529	.0685	.0268	.0093	.0029	.0008	.0002	.0000	.0000
	2	.0144	.1787	.2852	.2428	.1540	.0803	.0358	.0138	.0046	.0013	.0003	.0000

<i>n</i>	<i>x</i>	.01	.05	.10	.15	.20	.25	<i>p</i>	.30	.35	.40	.45	.50
	3	.0008	.0533	.1796	.2428	.2182	.1517	.0869	.0422	.0175	.0062	.0018	
	4	.0000	.0112	.0798	.1714	.2182	.2023	.1491	.0909	.0467	.0203	.0074	
	5	.0000	.0018	.0266	.0907	.1636	.2023	.1916	.1468	.0933	.0497	.0222	
	6	.0000	.0002	.0069	.0374	.0955	.1574	.1916	.1844	.1451	.0949	.0518	
	7	.0000	.0000	.0014	.0122	.0443	.0974	.1525	.1844	.1797	.1443	.0961	
	8	.0000	.0000	.0002	.0032	.0166	.0487	.0981	.1489	.1797	.1771	.1442	
	9	.0000	.0000	.0000	.0007	.0051	.0198	.0514	.0980	.1464	.1771	.1762	
	10	.0000	.0000	.0000	.0001	.0013	.0066	.0220	.0528	.0976	.1449	.1762	
	11	.0000	.0000	.0000	.0000	.0003	.0018	.0077	.0233	.0532	.0970	.1442	
	12	.0000	.0000	.0000	.0000	.0000	.0004	.0022	.0083	.0237	.0529	.0961	
	13	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0024	.0085	.0233	.0518	
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0082	.0222	
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0022	.0074	
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018	
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
20	0	.8179	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000	
	1	.1652	.3774	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000	
	2	.0159	.1887	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002	
	3	.0010	.0596	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011	
	4	.0000	.0133	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046	
	5	.0000	.0022	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148	
	6	.0000	.0003	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370	
	7	.0000	.0000	.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739	
	8	.0000	.0000	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201	
	9	.0000	.0000	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602	
	10	.0000	.0000	.0000	.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762	
	11	.0000	.0000	.0000	.0000	.0005	.0030	.0120	.0336	.0710	.1185	.1602	
	12	.0000	.0000	.0000	.0000	.0001	.0008	.0039	.0136	.0355	.0727	.1201	
	13	.0000	.0000	.0000	.0000	.0000	.0002	.0010	.0045	.0146	.0366	.0739	
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0049	.0150	.0370	
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0013	.0049	.0148	
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0046	
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011	
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
25	0	.7778	.2774	.0718	.0172	.0038	.0008	.0001	.0000	.0000	.0000	.0000	
	1	.1964	.3650	.1994	.0759	.0236	.0063	.0014	.0003	.0000	.0000	.0000	
	2	.0238	.2305	.2659	.1607	.0708	.0251	.0074	.0018	.0004	.0001	.0000	
	3	.0018	.0930	.2265	.2174	.1358	.0641	.0243	.0076	.0019	.0004	.0001	
	4	.0001	.0269	.1384	.2110	.1867	.1175	.0572	.0224	.0071	.0018	.0004	
	5	.0000	.0060	.0646	.1564	.1960	.1645	.1030	.0506	.0199	.0063	.0016	
	6	.0000	.0010	.0239	.0920	.1633	.1828	.1472	.0908	.0442	.0172	.0053	
	7	.0000	.0001	.0072	.0441	.1108	.1654	.1712	.1327	.0800	.0381	.0143	
	8	.0000	.0000	.0018	.0175	.0623	.1241	.1651	.1607	.1200	.0701	.0322	
	9	.0000	.0000	.0004	.0058	.0294	.0781	.1336	.1635	.1511	.1084	.0609	
	10	.0000	.0000	.0000	.0016	.0118	.0417	.0916	.1409	.1612	.1419	.0974	
	11	.0000	.0000	.0000	.0004	.0040	.0189	.0536	.1034	.1465	.1583	.1328	
	12	.0000	.0000	.0000	.0000	.0012	.0074	.0268	.0650	.1140	.1511	.1550	
	13	.0000	.0000	.0000	.0000	.0003	.0025	.0115	.0350	.0760	.1236	.1550	
	14	.0000	.0000	.0000	.0000	.0000	.0007	.0042	.0161	.0434	.0867	.1328	
	15	.0000	.0000	.0000	.0000	.0000	.0002	.0013	.0064	.0212	.0520	.0974	

<i>n</i>	<i>x</i>	.01	.05	.10	.15	.20	.25	<i>p</i>	.30	.35	.40	.45	.50	
30	16	.0000	.0000	.0000	.0000	.0000	.0000	.0004	.0021	.0088	.0266	.0609		
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0031	.0115	.0322		
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0009	.0042	.0143	
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0013	.0053	
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0016	
	21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004	
	22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
	0	.7397	.2146	.0424	.0076	.0012	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.2242	.3389	.1413	.0404	.0093	.0018	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0328	.2586	.2277	.1034	.0337	.0086	.0018	.0003	.0003	.0000	.0000	.0000	.0000
	3	.0031	.1270	.2361	.1703	.0785	.0269	.0072	.0015	.0015	.0003	.0000	.0000	.0000
	4	.0002	.0451	.1771	.2028	.1325	.0604	.0208	.0056	.0056	.0012	.0002	.0000	.0000
	5	.0000	.0124	.1023	.1861	.1723	.1047	.0464	.0157	.0157	.0041	.0008	.0001	.0001
	6	.0000	.0027	.0474	.1368	.1795	.1455	.0829	.0353	.0353	.0115	.0029	.0006	.0006
	7	.0000	.0005	.0180	.0828	.1538	.1662	.1219	.0652	.0652	.0263	.0081	.0019	.0019
	8	.0000	.0001	.0058	.0420	.1106	.1593	.1501	.1009	.1009	.0505	.0191	.0055	.0055
	9	.0000	.0000	.0016	.0181	.0676	.1298	.1573	.1328	.1328	.0823	.0382	.0133	.0133
	10	.0000	.0000	.0004	.0067	.0355	.0909	.1416	.1502	.1502	.1152	.0656	.0280	.0280
	11	.0000	.0000	.0001	.0022	.0161	.0551	.1103	.1471	.1471	.1396	.0976	.0509	.0509
	12	.0000	.0000	.0000	.0006	.0064	.0291	.0749	.1254	.1254	.1474	.1265	.0806	.0806
	13	.0000	.0000	.0000	.0001	.0022	.0134	.0444	.0935	.0935	.1360	.1433	.1115	.1115
	14	.0000	.0000	.0000	.0000	.0007	.0054	.0231	.0611	.0611	.1101	.1424	.1354	.1354
	15	.0000	.0000	.0000	.0000	.0002	.0019	.0106	.0351	.0351	.0783	.1242	.1445	.1445
	16	.0000	.0000	.0000	.0000	.0000	.0006	.0042	.0177	.0177	.0489	.0953	.1354	.1354
	17	.0000	.0000	.0000	.0000	.0000	.0002	.0015	.0079	.0079	.0269	.0642	.1115	.1115
18	.0000	.0000	.0000	.0000	.0000	.0000	.0005	.0031	.0031	.0129	.0379	.0806	.0806	
19	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0010	.0010	.0054	.0196	.0509	.0509	
20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0003	.0020	.0088	.0280	.0280	
21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0006	.0034	.0133	.0133	
22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0055	.0055	
23	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0019	.0019	
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0006	
25	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	

*Example: $P(X = 3, n = 5, p = 0.30) = 0.1323$.

APPENDIX 2

Poisson Distribution

Values of $e^{-\lambda}$

λ	$e^{-\lambda}$	λ	$e^{-\lambda}$
0.0	1.00000	2.5	.08208
0.1	.90484	2.6	.07427
0.2	.81873	2.7	.06721
0.3	.74082	2.8	.06081
0.4	.67032	2.9	.05502
0.5	.60653	3.0	.04979
0.6	.54881	3.2	.04076
0.7	.49659	3.4	.03337
0.8	.44933	3.6	.02732
0.9	.40657	3.8	.02237
1.0	.36788	4.0	.01832
1.1	.33287	4.2	.01500
1.2	.30119	4.4	.01228
1.3	.27253	4.6	.01005
1.4	.24660	4.8	.00823
1.5	.22313	5.0	.00674
1.6	.20190	5.5	.00409
1.7	.18268	6.0	.00248
1.8	.16530	6.5	.00150
1.9	.14957	7.0	.00091
2.0	.13534	7.5	.00055
2.1	.12246	8.0	.00034
2.2	.00180	8.5	.00020
2.3	.10026	9.0	.00012
2.4	.09072	10.0	.00005



STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

$$\text{Population mean, } \mu = \frac{\sum x}{N}$$

$$\text{Sample mean, } \bar{x} = \frac{\sum x}{n}$$

$$\text{Median, } M_e = \frac{n+1}{2}$$

Grouped data

$$\text{Population mean, } \mu = \frac{\sum fx}{N}$$

$$\text{Sample mean, } \bar{x} = \frac{\sum fx}{n}$$

$$\text{Median, } M_e = L_m + \frac{\left(\frac{n}{2} - F\right)C}{f_m}$$

$$\text{Mode, } M_o = L_m + \frac{(d_1) c}{d_1 + d_2}$$

MEASURES OF DISPERSION

Ungrouped data

$$\text{Population average deviation, } AD = \frac{\sum |x - \mu|}{N}$$

$$\text{Sample average deviation, } AD = \frac{\sum |x - \bar{x}|}{n}$$

$$\text{Population variance } = \sigma^2 = \frac{1}{N} (\sum x^2 - \frac{1}{N} (\sum x)^2)$$

$$\text{Sample variance } = s^2 = \frac{1}{n-1} (\sum x^2 - \frac{1}{n} (\sum x)^2)$$

$$\text{Population standard deviation, } \sigma = \sqrt{\frac{1}{N} (\sum x^2 - \frac{1}{N} (\sum x)^2)}$$

$$\text{Sample standard deviation, } s = \sqrt{\frac{1}{n-1} (\sum x^2 - \frac{1}{n} (\sum x)^2)}$$

Grouped data

Range = maximum value – minimum value

$$\text{Percentile, } P_k = L_p + \frac{\left(\frac{kn}{100} - F\right)C}{f_p}$$

$$\text{Decile, } D_x = L_d + \frac{\left(\frac{xn}{10} - F\right)C}{f_d}$$

$$\text{Lower quartile, } Q_1 = L_q + \frac{\left(\frac{n}{4} - F\right)C}{f_q}$$

$$\text{Upper quartile } Q_3 = L_q + \frac{\left(\frac{3n}{4} - F\right)C}{f_q}$$

$$\text{Interquartile range} = Q_3 - Q_1$$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2}$$

$$\text{Population average deviation, AD} = \frac{\sum f|x - \mu|}{N}$$

$$\text{Sample average deviation, AD} = \frac{\sum f|x - \bar{x}|}{n}$$

$$\text{Population variance, } \sigma^2 = \frac{1}{N} (\sum fx^2 - \frac{1}{N} (\sum fx)^2)$$

$$\text{Sample variance} = s^2 = \frac{1}{n-1} (\sum fx^2 - \frac{1}{n} (\sum fx)^2)$$

$$\text{Population standard deviation, } \sigma = \sqrt{\frac{1}{N} (\sum fx^2 - \frac{1}{N} (\sum fx)^2)}$$

$$\text{Sample standard deviation, } s = \sqrt{\frac{1}{n-1} (\sum fx^2 - \frac{1}{n} (\sum fx)^2)}$$

$$\text{Population coefficient of variation, CV} = \frac{\sigma}{\mu} \times 100\%$$

$$\text{Sample coefficient of variation, CV} = \frac{s}{\bar{x}} \times 100\%$$

SHAPE OF FREQUENCY DISTRIBUTIONS

$$\text{Population skewness, Sk} = \frac{3(\mu - \text{median})}{\sigma} \text{ or } \frac{(\mu - \text{mode})}{\sigma}$$

$$\text{Sample skewness, Sk} = \frac{3(\bar{x} - \text{median})}{s} \text{ or } \frac{(\bar{x} - \text{mode})}{s}$$

$$\text{Population kurtosis} = \frac{\sum f(X - \mu)^4}{\sigma^4}$$

$$\text{Sample kurtosis} = \frac{\sum f(X - \bar{x})^4}{s^4}$$

BASIC PROBABILITY CONCEPTS

i. **Classical Method of Assigning Probabilities:** $P(E) = \frac{ne}{N}$

ii. **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

iii. **Complementary rule:** $P(A') = 1 - P(A)$

iv. **Addition rule**

a) If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

- b) If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$
- v. **Multiplication rule**
- a) If X, Y are dependent (conditional probability),
 $P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$
- b) If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$
- vi. **Law of Conditional Probability**, $P(X | Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y | X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	Mean	Variance
Binomial distribution $P(x) = \frac{n!}{x!(n-x)!} p^x (q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Standard normal distribution $Z = \frac{x - \mu}{\sigma}$	0	1

CONFIDENCE INTERVALS

Mean of a single normal population

- i. If sample size is small (i.e., $n < 30$) and σ^2 Unknown

$$\bar{x} - t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2} (n - 1) \frac{s}{\sqrt{n}}$$

- ii. If sample size is large and σ^2 unknown

$$\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

- iii. If variance, σ^2 is known

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

- i. If when σ^2 is unknown and sample size is small ($n < 30$), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

- ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}} = Z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

- iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal

random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H ₀ if
H ₀ : μ = μ ₀ H ₁ : μ ≠ μ ₀	Z -distribution	Z > Z _{α/2} Z < - Z _{α/2}
	T - distribution	Z > Z _{α/2} T < - t _{α/2} (n-1)
H ₀ : μ ≤ μ ₀ H ₁ : μ > μ ₀	Z-distribution	Z > Z _α
	T-distribution	T > t _α (n-1)
H ₀ : μ ≥ μ ₀ H ₁ : μ < μ ₀	Z-distribution	Z < - Z _α
	T-distribution	T < - t _α (n-1)

Chi-square tests: $\chi^2_{\text{calc}} = \sum \frac{(f_o - f_e)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, $\text{Cov}(X, Y) = \frac{\sum(XY)}{n} - \bar{X} \bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n \sum XY - \sum X \sum Y}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n \sum XY - \sum X \sum Y}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}} \right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: $Y = a + bX$

$$b = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b \bar{X}$$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$

- ii. Rule 1: The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c .
- iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
- iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
- v. Rule 4: The chain rule. If y is a function of u , which is itself a function of x , then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

- vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

- viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{f_x}{f_y}$

Simple interest: $FV = \text{Principal} \times \text{Rate} \times \text{Time}$

Compound interest: $FV = A(1+i)^n$

Future Value of an Ordinary Annuity: $FV = R \left[\frac{(1+i)^n - 1}{i} \right]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$

Regular payment: $R = \frac{PVi}{1 - (1+i)^{-n}}$

Remaining Balance: $B = R \left[\frac{1 - (1+i)^{-(n-k)}}{i} \right]$

Net Present Value: $NPV = \sum_{j=0}^n \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + \left[\left(\frac{NPV_a}{NPV_a - NPV_b} \right) (b-a) \right] \%$

40/20 Am