ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS

DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

EXAMINATION PAPER

COURSE CODE	:	CBM122
COURSE TITLE	:	Quantitative Analysis for Business
SPECIAL REQUIREMENTS	:	Statistical Tables Formulae Sheets Graph Paper
DURATION	:	3 Hours
LEVEL	:	1.2
DATE	:	31 JUL 2024

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets [ ]
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR INSTRUCTS YOU.**

QUESTION ONE

a. The ABC Manufacturing Company makes electric wiring, which it sells to contractors in the construction industry. Approximately 900 electric contractors purchase wire from ABC annually. ABC's director of marketing wants to determine electric contractors' satisfaction with ABC's wire. He developed a questionnaire that yields a satisfaction score between 10 and 50 for participant responses. A random sample of 35 of the 900 contractors is asked to complete a satisfaction survey. The satisfaction scores for the 35 participants are averaged to produce a mean satisfaction score.

i. What is the population for this study? **[2 marks]**

ii. What is the sample for this study? **[2 marks]**

iii. What is the statistic for this study? **[2 marks]**

iv. What would be a parameter for this study? **[2 marks]**

b. The frequency distribution shown represents the number of farms per province for the 50 provinces based on information from the Department of Agriculture.

Number of Farms per provinces	**Frequency**
0– under 200	16
200– under 400	11
400– under 600	11
600– under 800	6
800 – under 1000	4
1000– under 1200	2

Required

i. Determine the average number of farms and standard deviation per state from these data. **[10 marks]**

ii. The mean computed from the original ungrouped data was 417.96 and the standard deviation was 388.56. Compare and contrast the answers for grouped data and ungrouped data. Why might they differ? **[7 marks]**

QUESTION TWO

a. A study by Peters Associates for the World Stock Market revealed that 43% of all adults are stockholders. In addition, the study determined that 75% of all adult stockholders have some college education. Suppose 37% of all adults have some college education. An adult is randomly selected.

i. What is the probability that the adult owns stock and has some college education? **[5 marks]**

ii. What is the probability that the adult owns stock or has some college education? **[5 marks]**

b. According to the National Environmental Program and World Health Organization, air pollution standards for particulate matter are exceeded an average of 5.6 days in every three-week period. Assume that the distribution of number of days exceeding the standards per three-week period is Poisson distributed. What is the probability that the standard is not exceeded on any day during a three-week period? **[5 marks]**

c. According to Student Monitor Application, the average cumulated college student loan debt for a graduating student is $25760. Assume that the standard deviation of such student loan debt is $5684. Thirty percent of these graduating students owe more than what amount? **[10 marks]**

QUESTION THREE

a. Suppose the age distribution in a city is as follows.

Age	**Percentage**
Under 18	22%
18–25	18%
26–50	36%
51–65	10%
Over 65	14%

A researcher is conducting proportionate stratified random sampling with a sample size of 250. Approximately how many people should he sample from each stratum? **[10 marks]**

b. According to the Bureau of Labor Statistics, the average weekly earnings of a production worker in 2017 were $424.20. Suppose a labor researcher wants to test to determine whether this figure is still accurate today. The researcher randomly selects 54 production workers from across the country and obtains a representative earnings statement for one week from each. The resulting sample average is $432.69. Assuming a population standard deviation of $33.90, and a 5% level of significance, determine whether the mean weekly earnings of a production worker have changed. **[15 marks]**

QUESTION FOUR

Is it possible to predict the annual number of business bankruptcies by the number of firm births (business starts) in the country? The following data published by the Small Business Administration, are pairs of the number of business bankruptcies and the number of firm births for a six-year period.

Business Bankruptcies	Firm Births
34.3	58.1
35.0	55.4
38.5	57.0
40.1	58.5
35.5	57.4
37.9	58.0

Required

a. Using a scatter plot, comment on the relationship between business bankruptcies by the number of firm births. **[5 marks]**

b. Use these data to develop the equation of the regression model to predict the number of business bankruptcies by the number of firm births. **[6 marks]**

c. Discuss the slope and *y*-intercept of the model. **[4 marks]**

d. Compute the coefficient of determination and comment. **[5 marks]**

e. Compute the correlation coefficient and comment. **[5 marks]**

QUESTION FIVE

a. Ron is buying a machine costing $120000 for a small-scale business. In order to run it he will spend $85000 in the first year, $30000 in the second, $15000 in the third year and a further $15000 in the fourth year. Revenues expected from the business are $80000, $120000, $100000 and \$65000 in the 1st, 2nd, 3rd, 4th years respectively. If the cost of capital is at 10% per annum, advise Ron as to whether the business is viable. **[15 marks]**

b. Your firm has recently started to give economic advice to your clients. Acting as a consultant you have estimated the demand curve of a client's firm to be:

$$P = 200 - 8x, \text{ where } x \text{ is output and } P \text{ is price.}$$

Investigation of the client firm's cost (TC) profile shows that total cost is given by:

$$TC = \frac{1}{3}x^3 - 14x^2 + 211x + 10$$

i. Using the methods of differentiation, find the turning points of the firm's profit curve and say whether these point(s) are maxima or minima. **[6 marks]**

ii. Calculate the maximum profit. **[4 marks]**

THE END

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

Population mean, $\mu = \frac{\sum x}{N}$

Sample mean, $\bar{x} = \frac{\sum x}{n}$

Median, $M_e = \frac{n+1}{2}$

Grouped data

Population mean, $\mu = \frac{\sum fx}{N}$

Sample mean, $\bar{x} = \frac{\sum fx}{n}$

Median, $Me = L_m + \frac{(\frac{n}{2} - F)C}{fm}$

Mode, $Mo = L_m + \frac{(d_1)\,c}{d_1 + d_2}$

MEASURES OF DISPERSION

Ungrouped data

Population average deviation, $AD = \frac{\sum|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum|x-\bar{x}|}{n}$

Population variance $= \sigma^2 = \frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)}$

Grouped data

Range = maximum value – minimum value

Percentile, $P_k = L_p + \frac{(\frac{kn}{100} - F)C}{fp}$

Decile, $D_x = L_d + \frac{(\frac{xn}{10} - F)C}{fd}$

Lower quartile, $Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{fq}$

Upper quartile $Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{fq}$

Interquartile range = $Q_3 - Q_1$

Quartile deviation = $\frac{Q_3 - Q_1}{2}$

Population average deviation, $AD = \frac{\sum f|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum f|x-\bar{x}|}{n}$

Population variance, $\sigma^2 = \frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, $CV = \frac{\sigma}{\mu} \times 100\%$

Sample coefficient of variation, $CV = \frac{s}{\bar{x}} \times 100\%$

SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, $Sk = \frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, $Sk = \frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis = $\frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis = = $\frac{\sum f(X-\bar{x})^4}{s^4}$

BASIC PROBABILITY CONCEPTS

i. **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

ii. **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

iii. **Complementary rule:** $P(A') = 1 - P(A)$

iv. **Addition rule**

a) If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

b) If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

v. **Multiplication rule**

a) If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

b) If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

vi. **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	**Mean**	**Variance**
Binomial distribution $P(x) = \frac{n!}{x!\,(n-x)!} p^x (q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	**λ**	**λ**
Standard normal distribution	**0**	**1**

$z = \frac{x - \mu}{\sigma}$		

CONFIDENCE INTERVALS

Mean of a single normal population

i. **If sample size is small (i.e., n<30) and σ^2 Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and σ^2 unknown**

$$\bar{x} - Z_{\alpha/2}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm Z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, σ^2 is known**

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when σ^2 is unknown and sample size is small (n<30), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	Distribution	Reject H_0 if
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T - distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \le \mu_0$ $H_1: \mu > \mu_0$	Z-distribution T-distribution	$Z > Z_\alpha$ $T > t_\alpha (n-1)$
$H_0: \mu \ge \mu_0$ $H_1: \mu < \mu_0$	Z-distribution T-distribution	$Z < -Z_\alpha$ $T < -t_\alpha (n-1)$

Chi-square tests: $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, Cov(X, Y) $= \frac{\sum(XY)}{n} - \bar{X}\,\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}\right)^2$

TIME SERIES ANALYSIS

Fitted Trend Line: $Y = a + bX$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$a = \bar{Y} - b\,\bar{X}$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$
viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

Simple interest: FV = Principal x Rate x Time

Compound interest: $FV = A(1+i)^n$

Future Value of an Ordinary Annuity: $FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

Remaining Balance: $B = R\left[\frac{1-(1+i)^{-(n-X)}}{i}\right]$

Net Present Value: $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + [(\frac{NPV_a}{NPV_a - NPV_b})(b-a)]\ \%$

7 Am