ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS

DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

EXAMINATION PAPER

COURSE CODE	:	CBM122
COURSE TITLE	:	Quantitative Analysis for Business
SPECIAL REQUIREMENTS	:	Statistical Tables Formulae Sheets Graph Paper
DURATION	:	3 Hours
LEVEL	:	1.2
DATE	:	08 APR 2024

INSTRUCTIONS TO CANDIDATES:

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets [ ]
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR INSTRUCTS YOU.**

QUESTION ONE

As a researcher, you are tasked to a survey.

a. Select a research topic of your choice and clearly state three (3) research objectives and three (3) questions. **[7 marks]**

b. To do conduct this survey, you would like to select a sample of participants. Give five reasons why you opted to use a sample in the study instead of the whole population of preferred participants? **[5 marks]**

c. Discuss with justification an appropriate sampling method for the study. **[5 marks]**

d. Design a comprehensive questionnaire that addresses the stated research objectives and questions. **[8 marks]**

QUESTION TWO

a. Suppose short-run inverse demand in a monopolistically competitive market is represented by:

$$P = 18 - 0.2Q$$

Cost is given by:

$$TC = 300 + 2Q + 0.05Q^2 + 0.01Q^3$$

Given these demand and cost conditions, what price, output, and profits result in the short run? **[20 Marks]**

b. A farmer purchased a John Deere combine for $400000. The equipment dealership sets up a financing plan to allow for end-of-quarter payments for the next two years at 6% compounded quarterly. Calculate the periodic payment. **[5 marks]**

QUESTION THREE

a. A trucking firm is suspicious of the claim that the average life span of certain tyres is at least 82000km. The standard deviation is known to be 13450 km. To test the claim, the firm put 40 of these tyres on its trucks and got a mean life span of 72463km.

Required

At the 0.01 level of significance, perform an appropriate test. **[15 marks]**

b. Bemba conducted a national survey of small-business owners to determine the challenges for growth for their businesses. The top challenge, selected by 46% of the small business owners, was the economy. A second finding was qualified workers (37%). Suppose 15% of the small-business owners selected both the economy and finding qualified workers as challenges for growth. A small-business owner is randomly selected. What is the probability that the owner believes the economy is a challenge for growth if the owner believes that finding qualified workers is a challenge for growth? **[5 marks]**

c. According to a report by Chipo, the average monthly household cellular phone bill is $60. Suppose local monthly household cell phone bills are normally distributed with a standard deviation of $11.35. What is the probability that a randomly selected monthly cell phone bill is greater than $80? **[5 marks]**

QUESTION FOUR

The speed of cars recorded at a speed trap in Harare by a detector between 9:00am and 12:00pm on a Public Holiday morning was summarized in the table below:

Speed in Km/h	35 - < 45	45 - < 55	55 - < 65	65 - < 75
Frequency	33	40	30	20

a. Present the data on a Histogram and use the presentation to estimate the modal speed. **[5 marks]**

b. Compute and interpret the mean, mode, median and coefficient of variation. **[8 marks]**

c. Write a brief report commenting on the statistics computed in (ii) above. Be sure to include the skewness of the speed, the variability of the speeds as well as the reliability of the measures of central location. **[6 marks]**

d. What strategic recommendations can you give the Police Superintendent for Harare? **[6 marks]**

QUESTION FIVE

A company in Harare CBD keeps extensive records on its new sales people on the premise that sales should increase with experience. A random sample of ten new sales people produced the data on experience and sales shown in the table below:

Sales person	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Months on job	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5
Monthly sales	25	31	27	28	36	35	32	34	33	37

Required

a. Using a scatter plot, can sales be predicted by the months on the job? **[5 marks]**

b. Compute the regression equation and hence find the line of best fit. **[6 marks]**

c. Explain the significance of the ecoefficiencies of the regression equation in (b) above. **[4 marks]**

d. Compute the coefficient of determination and comment **[5 marks]**

e. Compute the correlation coefficient and comment **[5 marks]**

THE END

STATISTICAL FORMULAS

MEASURES OF CENTRAL TENDENCY

Ungrouped data

Population mean, $\mu = \frac{\sum x}{N}$

Sample mean, $\bar{x} = \frac{\sum x}{n}$

Median, $M_e = \frac{n+1}{2}$

Grouped data

Population mean, $\mu = \frac{\sum fx}{N}$

Sample mean, $\bar{x} = \frac{\sum fx}{n}$

Median, $Me = L_m + \frac{(\frac{n}{2} - F)C}{fm}$

Mode, $Mo = L_m + \frac{(d_1)\ c}{d_1 + d_2}$

MEASURES OF DISPERSION

Ungrouped data

Population average deviation, $AD = \frac{\sum|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum|x-\bar{x}|}{n}$

Population variance $= \sigma^2 = \frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum x^2 - \frac{1}{N}(\sum x)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum x^2 - \frac{1}{n}(\sum x)^2)}$

Grouped data

Range = maximum value – minimum value

Percentile, $P_k = L_p + \frac{(\frac{kn}{100} - F)C}{fp}$

Decile, $D_x = L_d + \frac{(\frac{xn}{10} - F)C}{fd}$

Lower quartile, $Q_1 = L_q + \frac{(\frac{n}{4} - F)C}{fq}$

Upper quartile $Q_3 = L_q + \frac{(\frac{3n}{4} - F)C}{fq}$

Interquartile range = $Q_3 - Q_1$

Quartile deviation = $\frac{Q_3 - Q_1}{2}$

Population average deviation, $AD = \frac{\sum f|x-\mu|}{N}$

Sample average deviation, $AD = \frac{\sum f|x-\bar{x}|}{n}$

Population variance, $\sigma^2 = \frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)$

Sample variance $= s^2 = \frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)$

Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, $CV = \frac{\sigma}{\mu} \times 100\%$

Sample coefficient of variation, $CV = \frac{s}{\bar{x}} \times 100\%$

SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, $Sk = \frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, $Sk = \frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis = $\frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis = = $\frac{\sum f(X-\bar{x})^4}{s^4}$

BASIC PROBABILITY CONCEPTS

i. **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

ii. **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

iii. **Complementary rule:** $P(A') = 1 - P(A)$

iv. **Addition rule**

a) If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

b) If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

v. **Multiplication rule**

a) If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

b) If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

vi. **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

PROBABILITY DISTRIBUTION

Probability distribution	**Mean**	**Variance**
Binomial distribution $P(x) = \frac{n!}{x!\,(n-x)!} p^x (q)^{n-x}$	np	npq
Poisson distribution $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	**λ**	**λ**
Standard normal distribution $z = \frac{x - \mu}{\sigma}$	**0**	**1**

CONFIDENCE INTERVALS

Mean of a single normal population

i. **If sample size is small (i.e., n<30) and σ^2 Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and σ^2 unknown**

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, σ^2 is known**

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when σ^2 is unknown and sample size is small (n<30), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when σ^2 is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when σ^2 is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

Hypothesis to be tested	**Distribution**	**Reject H_0 if**
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	Z -distribution	$Z > Z_{\alpha/2}$ $Z < -Z_{\alpha/2}$
	T - distribution	$Z > Z_{\alpha/2}$ $T < -t_{\alpha/2}(n-1)$
$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	Z-distribution T-distribution	$Z > Z_\alpha$ $T > t_\alpha(n-1)$
$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	Z-distribution T-distribution	$Z < -Z_\alpha$ $T < -t_\alpha(n-1)$

Chi-square tests: $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$

SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, $Cov(X, Y) = \frac{\sum(XY)}{n} - \bar{X}\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = (\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}})^2$

TIME SERIES ANALYSIS

Fitted Trend Line: $Y = a + bX$

$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$

$a = \overline{Y} - b\,\overline{X}$

BUSINESS CALCULATIONS

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \text{x} \frac{du}{dx}$$

vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$
viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

Simple interest: FV = Principal x Rate x Time

Compound interest: $FV = A(1 + i)^n$

Future Value of an Ordinary Annuity: $FV = R\,[\frac{(1+i)^n - 1}{i}]$

Regular deposit: $R = \frac{FVi}{(1+i)^n - 1}$

Present Value of an Ordinary Annuity: $PV = R\,[\frac{1-(1+i)^{-n}}{i}]$

Regular payment: $R = \frac{PVi}{1-(1+i)^{-n}}$

Remaining Balance: $B = R[\frac{1-(1+i)^{-(n-X)}}{i}]$

Net Present Value: $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

Internal Rate of Return: $IRR = a + [(\frac{NPV_a}{NPV_a - NPV_b})(b-a)]\ \%$

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