ZIMBABWE EZEKIEL GUTI UNIVERSITY

FACULTY OF LAW, BUSINESS INTELLIGENCE AND ECONOMICS

DEPARTMENT OF ECONOMICS, MARKETING AND ENTREPRENEURSHIP

# EXAMINATION PAPER

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|---|---|---|
| COURSE CODE | : | CBM122 |
| COURSE TITLE | : | Quantitative Analysis for Business |
| SPECIAL REQUIREMENTS | : | Statistical Tables<br>Formulae Sheets<br>Graph Paper |
| DURATION | : | 3 Hours |
| LEVEL | : | 1.2 |
| DATE | : | 14 JUN 2024 |

**INSTRUCTIONS TO CANDIDATES:**

1. No cell phones are allowed in the examination venue.
2. Answer any **FOUR (4)** questions.
3. The number of marks for each question or part question is shown in brackets [ ]
4. Use of non-programmable calculators is allowed.
5. Show all your workings in order to gain full marks.
6. Begin each answer on a new page.
7. **DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR INSTRUCTS YOU.**

## QUESTION ONE

**a.** The following data represent the afternoon high temperatures for 50 construction days during a year in Country X.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 42 | 70 | 64 | 47 | 66 | 69 | 73 | 38 | 48 | 25 | 55 | 85 |
| 10 | 24 | 45 | 31 | 62 | 47 | 63 | 84 | 16 | 40 | 81 | 15 |
| 35 | 17 | 40 | 36 | 44 | 17 | 38 | 79 | 35 | 36 | 23 | 64 |
| 75 | 53 | 31 | 60 | 31 | 38 | 52 | 16 | 81 | 12 | 61 | 43 |
| 30 | 33 | | | | | | | | | | |

i. Construct a frequency distribution for the data using five class intervals. **[5 marks]**

ii. Construct a frequency distribution for the data using 10 class intervals. **[5 marks]**

iii. Examine the results of (a) and (b) and comment on the usefulness of the frequency distribution in terms of temperature summarization capability. **[5 marks]**

**b.** Below is a distribution of randomly selected farmers in Mashonaland West and their respective production outputs:

| **Classes** | **0 -< 10** | **10 -< 20** | **20 - < 30** | **30 - < 40** | **40 - < 50** |
|---|---|---|---|---|---|
| **Less than cumulative frequencies** | 210 | 344 | 422 | 494 | 540 |

**Required:**
Calculate the mean, median and mode and provide strategic recommendations to the Ministry of Agriculture basing on the nature (skewness) of the distribution. **[10 marks]**

## QUESTION TWO

**a.** Suppose you work for a large firm that has 20000 employees. The CEO calls you in and asks you to determine employee attitudes toward the company. She is willing to commit $100000 to this project. What are the advantages of taking a sample versus conducting a census? **[10 marks]**

**b.** According to the Bureau of Labor Statistics, 75% of the women 25 through 49 years of age participate in the labor force. Suppose 78% of the women in that age group are married. Suppose also that 61% of all women 25 through 49 years of age are married and are participating in the labor force. What is the probability that a randomly selected woman in that age group is neither married nor participating in the labor force? **[5 marks]**

c. In a study undertaken by Penny, 43% of women senior executives agreed or strongly agreed that a lack of role models was a barrier to their career development. In addition, 46% agreed or strongly agreed that gender-based stereotypes were barriers to their career advancement. Suppose 77% of those who agreed or strongly agreed that gender-based stereotypes were barriers to their career advancement agreed or strongly agreed that the lack of role models was a barrier to their career development. If one of these female senior executives is randomly selected, determine the probability that the senior executive does not agree or strongly agree that gender-based stereotypes were barriers to her career development given that she does agree or strongly agree that the lack of role models was a barrier to her career development? **[5 marks]**

d. According to the Revenue Authority, income tax returns one year averaged $1332 in refunds for taxpayers. One explanation of this figure is that taxpayers would rather have the government keep back too much money during the year than to owe it money at the end of the year. Suppose the average amount of tax at the end of a year is a refund of $1332, with a standard deviation of $725. Assume that amounts owed or due on tax returns are normally distributed. What proportion of tax returns show a refund greater than $2000? **[5 marks]**

## QUESTION THREE

a. Life insurance experts have been claiming that the average worker in the city of Bindura has no more than $25000 of personal life insurance. An insurance researcher believes that this is not true and sets out to prove that the average worker in Bindura has more than $25000 of personal life insurance. To test this claim, she randomly samples 100 workers in Bindura and interviews them about their personal life insurance coverage. She discovers that the average amount of personal life insurance coverage for this sample group is $26650. The population standard deviation is $12000. Determine at 5% level of significance, whether the test shows enough evidence to reject the null hypothesis posed by the salesperson. **[15 marks]**

b. Hwiza farm are making brickets used as fuel-wood for the community around as part of their social responsibility. The total cost of producing x brickets is defined by the function $C(x) = 20 + 3x$. The farm discovers that the selling price is given by the function: $P = 32 - 2x$. The manager wants you to help him determine:

   i. Output, which maximizes revenue and the maximum revenue. **[5 marks]**

   ii. Output, which maximizes profit and the maximum profit. **[5 marks]**

## QUESTION FOUR

A corporation owns several companies. The strategic planner for the corporation believes dollars spent on advertising can to some extent be a predictor of total sales dollars. As an aid in long-term planning, she gathers the following sales and advertising information from several of the companies for 2022.

| Advertising | Sales |
|---|---|
| 12.5 | 148 |
| 3.7 | 55 |
| 21.6 | 338 |
| 60.0 | 994 |
| 37.6 | 541 |
| 6.1 | 89 |
| 16.8 | 126 |
| 41.2 | 379 |

a. Plot the data and comment. **[5 marks]**

b. Develop the equation of the simple regression line to predict sales from advertising expenditures using these data. **[6 marks]**

c. Using the linear regression equation, interpret the coefficients. **[4 marks]**

d. Compute the coefficient of determination and comment. **[5 marks]**

e. Compute the correlation coefficient and comment. **[5 marks]**

## QUESTION FIVE

Your company uses a machine in its production department, which costs $44000 at the beginning of 2017. The machine will be replaced after five years usage by a new machine at the end of 2021. During the five years of operation of the machine it is estimated that the net cash inflows at the beginning of each year will be as follows:

| Year | 2018 | 2019 | 2020 | 2021 | 2022 |
|---|---|---|---|---|---|
| Net cash inflow ($) | 26000 | 30000 | 50000 | (25000) | (12000) |

***Required:***

a. If the machine is being purchased with a five-year loan, which is compounded annually at 6%, produce an amortization schedule for the loan. **[10 marks]**

b. If the $44000 debt, which is compounded semi-annually at 6%, is to be discharged in 2022 by a sinking fund method, under which annual deposits will be made into a fund paying 4% annually, produce the schedule for the sinking fund. **[10 marks]**

c. Calculate the net present value of the net cash flows over the five years of operation of the machine at the 15% discount rates. **[5 marks]**

**THE END**

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Population standard deviation, $\sigma = \sqrt{\frac{1}{N}(\sum fx^2 - \frac{1}{N}(\sum fx)^2)}$

Sample standard deviation, $s = \sqrt{\frac{1}{n-1}(\sum fx^2 - \frac{1}{n}(\sum fx)^2)}$

Population coefficient of variation, $CV = \frac{\sigma}{\mu} \times 100\%$

Sample coefficient of variation, $CV = \frac{s}{\bar{x}} \times 100\%$

## SHAPE OF FREQUENCY DISTRIBUTIONS

Population skewness, $Sk = \frac{3(\mu - \text{median})}{\sigma}$ or $\frac{(\mu - \text{mode})}{\sigma}$

Sample skewness, $Sk = \frac{3(\bar{x} - \text{median})}{s}$ or $\frac{(\bar{x} - \text{mode})}{s}$

Population kurtosis = $\frac{\sum f(X-\mu)^4}{\sigma^4}$

Sample kurtosis = = $\frac{\sum f(X-\bar{x})^4}{s^4}$

## BASIC PROBABILITY CONCEPTS

i. **Classical Method of Assigning Probabilities:** $P(E) = \frac{n_e}{N}$

ii. **Probability by Relative Frequency of Occurrence**

$$\frac{\text{Number of Times an Event Occurred}}{\text{Total Number of Opportunities for the Event to Occur}}$$

iii. **Complementary rule:** $P(A') = 1 - P(A)$

iv. **Addition rule**

a) If X, Y are non-mutually exclusive, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

b) If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$

v. **Multiplication rule**

a) If X, Y are dependent (conditional probability),

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

b) If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

vi. **Law of Conditional Probability,** $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y \mid X)}{P(Y)}$

## PROBABILITY DISTRIBUTION

| **Probability distribution** | **Mean** | **Variance** |
|---|---|---|
| Binomial distribution<br>$P(x) = \frac{n!}{x!\,(n-x)!} p^x (q)^{n-x}$ | np | npq |
| Poisson distribution<br>$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ | **λ** | **λ** |
| Standard normal distribution<br>$z = \frac{x - \mu}{\sigma}$ | **0** | **1** |

## CONFIDENCE INTERVALS

**Mean of a single normal population**

i. **If sample size is small (i.e., n<30) and $\sigma^2$ Unknown**

$$\bar{x} - t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} \text{ or simply } \bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$$

ii. **If sample size is large and $\sigma^2$ unknown**

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

iii. **If variance, $\sigma^2$ is known**

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or simply, } \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

## TESTING OF HYPOTHESIS

Tests on the mean of a single population

i. If when $\sigma^2$ is unknown and sample size is small (n<30), then the t Statistic is such that,

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

ii. If when $\sigma^2$ is known and sample size is large, then the z statistic is such that:

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}} = z = \frac{\bar{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}}$$

iii. If when $\sigma^2$ is unknown and sample size is large, then the t statistic is approximately a standard normal random variable such that: $z = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$

| **Hypothesis to be tested** | **Distribution** | **Reject $H_0$ if** |
|---|---|---|
| $H_0: \mu = \mu_0$<br>$H_1: \mu \neq \mu_0$ | Z -distribution | $Z > Z_{\alpha/2}$<br>$Z < -Z_{\alpha/2}$ |
| | T - distribution | $Z > Z_{\alpha/2}$<br>$T < -t_{\alpha/2}(n-1)$ |
| $H_0: \mu \leq \mu_0$<br>$H_1: \mu > \mu_0$ | Z-distribution<br>T-distribution | $Z > Z_\alpha$<br>$T > t_\alpha (n-1)$ |
| $H_0: \mu \geq \mu_0$<br>$H_1: \mu < \mu_0$ | Z-distribution<br>T-distribution | $Z < -Z_\alpha$<br>$T < -t_\alpha (n-1)$ |

**Chi-square tests:** $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$

## SIMPLE REGRESSION AND CORRELATION

Fitted linear model: $\hat{Y} = \hat{b}_0 + \hat{b}_1 X$

$$\hat{b}_1 = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Covariance, $Cov(X, Y) = \frac{\sum(XY)}{n} - \bar{X}\bar{Y}$

Correlation coefficient, $r = \sqrt{R^2} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Coefficient of determination, $R^2 = \left(\frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}\right)^2$

**TIME SERIES ANALYSIS**

Fitted Trend Line: $Y = a + bX$

$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$

$a = \overline{Y} - b\,\overline{X}$

**BUSINESS CALCULATIONS**

**Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Differentiation**

i. if $f(x) = x^n$ then $f'(x) = nx^{n-1}$; if $y = x^n$ then, $\frac{dy}{dx} = nx^{n-1}$
ii. Rule 1. The constant rule. If $h(x) = cf(x)$ then $h'(x) = cf'(x)$ for any constant c.
iii. Rule 2: The sum rule. If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
iv. Rule 3: The difference rule. If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
v. Rule 4: The chain rule. If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

vi. Rule 5: The product rule. If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
vii. Rule 6: The quotient rule. If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$
viii. Implicit differentiation: $\frac{dy}{dx} = -\frac{fx}{fy}$

**Simple interest:** FV = Principal x Rate x Time

**Compound interest:** $FV = A(1+i)^n$

**Future Value of an Ordinary Annuity:** $FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

**Regular deposit:** $R = \frac{FVi}{(1+i)^n - 1}$

**Present Value of an Ordinary Annuity:** $PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

**Regular payment:** $R = \frac{PVi}{1-(1+i)^{-n}}$

**Remaining Balance:** $B = R\left[\frac{1-(1+i)^{-(n-X)}}{i}\right]$

**Net Present Value:** $NPV = \sum_{j=0}^{n} \frac{R_j}{(1+i)^j}$

**Internal Rate of Return:** $IRR = a + \left[\left(\frac{NPV_a}{NPV_a - NPV_b}\right)(b-a)\right]\%$